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Modeling the History of Astronomy: Ptolemy, Copernicus, and Tycho

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Abstract

This paper describes a series of activities in which students investigate and use the Ptolemaic, Copernican, and Tychonic models of planetary motion. The activities guide students through using open source software to discover important observational facts, learn the necessary vocabulary, understand the fundamental properties of different theoretical models, and relate the theoretical models to observational data. After completing these activities, students can make observations of a fictitious solar system and use those observations to construct models for that system.

1. INTRODUCTION

If we are to shape student attitudes about science in an introductory astronomy course, then we must go beyond teaching the results of science and engage students in the scientific process (Wittman 2009; Duncan and Arthurs 2012). Students should have the opportunity to develop and test models of natural phenomena and even evaluate competing scientific models (Etikina, Warren, and Gentile 2006). One way to involve students in this kind of authentic scientific investigation is to have them investigate scientific models from the history of science (Matthews 1994). This paper describes a series of activities in which students investigate and use models that were important in the historical development of planetary astronomy. The models examined are those introduced by the Hellenistic astronomer Claudius Ptolemy in the second Century, by the Polish astronomer Nicolaus Copernicus in the mid-16th Century, and by the Danish astronomer Tycho Brahe in the late 16th Century. For more information on these theories and the history of planetary astronomy see Kuhn (1985), Crowe (1990), and Linton (2004).

The activities help students use open source software to discover important observational facts, learn the necessary vocabulary, understand the fundamental properties of different theoretical models, and relate the theoretical models to observational data. Once they understand the observations and models, students complete a series of projects in which they observe and model a fictitious solar system with four planets orbiting in circles around a central star. For this purpose, each student is given a different computer program that simulates the motion of the central star and three planets (one of the four planets is the observer's home planet) against a fixed background of stars. Most of the computer simulations were created with *Easy Java Simulations* (Esquembre 2013) and are part of the *Open Source Physics* collection (Open Source Physics 2013). All of the materials described in this paper have been collected in a shared folder in the *Open Source Physics* collection on the *ComPADRE* digital archive (see Timberlake 2013). Instructors can visit this web page to download the computer simulations as well as zip archives containing student activity handouts and student project handouts.

2. OBSERVING THE SKY

The Ptolemaic, Copernican, and Tychonic theories were attempts to model naked-eye observations of the night sky. Before exploring these theories, students make simulated observations (both qualitative and quantitative) of the night sky using the open-source planetarium program *Stellarium* (Stellarium 2013).

Observations of the stars lead students to their first astronomical model: the *Celestial Sphere*. The stars appear to move as though they are stuck on the surface of a giant sphere with Earth at the center. The Celestial Sphere rotates east to west once every *sidereal day* (23 h, 56 min) about a fixed axis. For the purpose of examining the motions of all other celestial bodies, the Celestial Sphere can be used as a fixed background. For example, students discover that the Sun drifts roughly eastward relative to the Celestial Sphere along a great circle path known as the *Ecliptic*. As a result of this drift, it takes slightly longer (24 h, or one *solar day*) for the Sun to complete a full rotation in our sky, and the Sun completes one full circuit along the Ecliptic in a sidereal year (about 365.25 solar days).

Students then observe the five visible planets: Mercury, Venus, Mars, Jupiter, and Saturn. They find that all five planets move relative to the Celestial Sphere. The planets move generally eastward and remain near the Ecliptic, although they can be above or below. The average time for a planet to complete one circuit along the Ecliptic is the planet's *sidereal period* (also known as the *zodiacal period*, since all planets move through the constellations of the Zodiac). Planets also move relative to the Sun. The *elongation* of a planet is the angle between the planet and the Sun on the sky. If a planet is at 0° elongation, it is said to be in *conjunction*, at 90° it is in *quadrature*, and at 180° it is in *opposition*. The Sun periodically moves eastward past each planet, and the time between one pass and the next is that planet's *synodic period*. Students are often surprised to discover that the planets occasionally move westward for a short time, in what is called *retrograde* motion, before resuming their eastward motion. The time between successive retrogrades is observed to be equal to the planet's synodic period.

In spite of these general characteristics, not all planets behave the same. Mercury and Venus are never more than 28° and 48°, respectively, from the Sun. They are in conjunction during the middle of their retrograde motion. These planets are known as *inferior planets*. Mars, Jupiter, and Saturn can attain any elongation. They are in opposition during the middle of their retrograde motion. These planets are known as *superior planets*. All planets appear somewhat brighter during retrograde, but Mars displays the greatest increase in brightness.

After exploring and measuring the real (simulated) night sky, students make observations of the night sky in their personalized solar system (see Figure 1). Students must determine the number of planets and classify each planet as inferior or superior. They must measure the sidereal year (T_{sy}), as well as the zodiacal period (T_z) and synodic period (T_s) of each planet. They must measure the maximum elongation (α) of each inferior planet and the time from opposition to quadrature (t_Q) for each superior planet. These measurements will be used later to construct models of their solar system.

Most students have little difficulty with determining the number of planets, and how many are inferior or superior. Errors in the measurement of the synodic period usually result from misunderstanding the definition, while errors in measuring the zodiacal period are more likely to be the result of averaging over too few cycles (or averaging incorrectly). Measuring α requires students to carefully monitor the changing elongation of the planet, while measuring t_Q requires students to carefully determine the moments of opposition and quadrature. Students who understand the operational definitions of these quantities, and who are willing to take the time to make careful measurements, usually get results that are good enough to construct accurate models for their solar system.

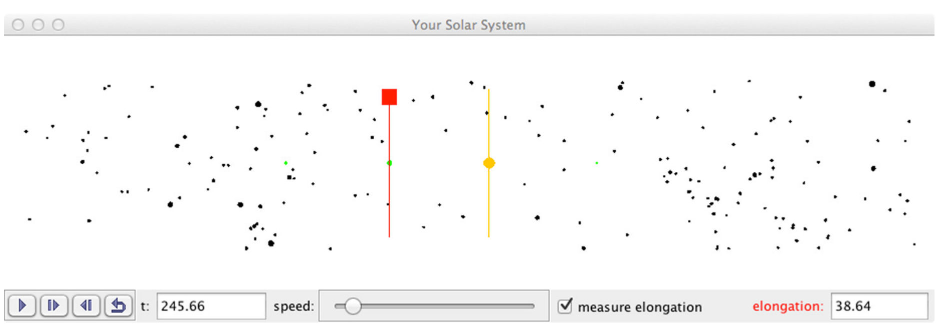

Figure 1. Modified screenshot of the fictitious solar system simulation. The simulation shows the night sky as seen from a fictitious planet located in a nearby solar system. The central star of the system is the yellow disk near the center. The three visible planets are shown in green. The red line can be placed (by dragging the attached square) to measure the elongation, or angle from the Sun, of any point on the sky. The elongation is given in degrees at the bottom right. In the real simulation, the stars and planets are white against a black background, making it difficult to distinguish planets from stars in a still frame.

3. PTOLEMAIC MODELING

The next set of activities helps students explore a simplified version of Ptolemy's model for planetary motions using the *Inferior Ptolemaic* and *Superior Ptolemaic* EJS programs (Timberlake 2013). In the simplified Ptolemaic model, each planet moves uniformly counterclockwise on a circle called the *epicycle*, while the center of the epicycle moves uniformly counterclockwise along the *deferent*, a larger circle centered on Earth. The Sun moves uniformly counterclockwise along a circle centered on Earth (see Note-1).

By working with the simulations, students discover that the period of the Sun's orbit must equal the sidereal year. They find that planets retrograde when the motion of the planet along the epicycle is in the direction opposite of the motion of the epicycle center along the deferent, which occurs when the planet is on the innermost part of the epicycle. The period of the epicycle center's motion around the deferent must equal the planet's zodiacal period. If the period of the planet's motion around the epicycle is measured relative to the deferent (so that a full period is measured from the time the planet crosses the deferent circle going outward until the next time it does so) then this epicycle period must equal the planet's synodic period.

Students also discover the main difference between the Ptolemaic theories for inferior and superior planets. The center of an inferior planet's epicycle must remain on the Earth-Sun line in order to keep the planet near the Sun in the sky. For superior planets, on the other hand, the line from the center of the epicycle to the planet must always be parallel to the Earth-Sun line in order to ensure that the planet always retrogrades at opposition.

Students can also determine the relative sizes of each planet's epicycle and deferent. For an inferior planet, the ratio of its epicycle radius to its deferent radius is given by

$$\frac{R_e}{R_d} = \sin\alpha, \tag{1}$$

where α is the planet's maximum elongation, as shown in Figure 2.

For a superior planet, this ratio can be computed by comparing the geometries at opposition and at quadrature, as shown in Figure 3. The result looks very similar to Eq. (1)

$$\frac{R_e}{R_d} = \sin\theta, \tag{2}$$

but note that α has been replaced by θ, an angle that is not observable (because there is nothing visible at the center of the epicycle). By examining the change between opposition and quadrature and recalling that all motions are uniform, students can show that

$$\theta = 90^\circ + 360^\circ\frac{t_Q}{T_z} - 360^\circ\frac{t_Q}{T_{sy}}, \tag{3}$$

where t_Q is the planet's time from opposition to quadrature, T_z is the planet's zodiacal period, and T_{sy} is the sidereal year.

After exploring the simplified Ptolemaic model of our solar system, students can develop a Ptolemaic model for their personalized fictitious solar system using the observational data they collected earlier. The most difficult part of this modeling for most students is the determination of the epicycle-to-deferent size ratios. Some students will want to find two separate numbers that they can divide to get a ratio, rather than using Eq. (1) and their measurement for maximum elongation. These students may calculate the ratio of the epicycle and deferent *periods*, thinking that this will give them the size ratio. Perhaps the most challenging part of this exercise is constructing a proper diagram for the Ptolemaic solar system: students may have trouble with the relative scaling of epicycle and deferent, or they may fail to place the planets in locations that conform to the Ptolemaic principles that link each planet's position to the Sun's position.

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[1]Note 1: In the full Ptolemaic model, the Sun's orbit and the deferent circles were not centered on Earth, and the epicycle centers did not move uniformly along the deferent. The theories for Mercury and Venus had additional complications.

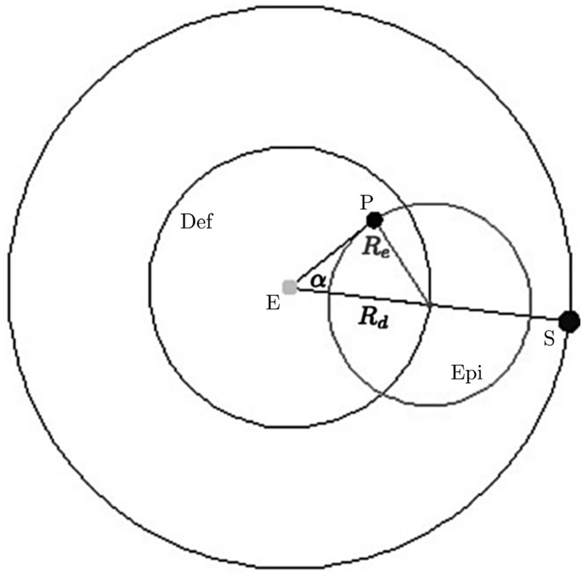

Figure 2. Ptolemaic geometry for an inferior planet at maximum elongation, modified from *Inferior Ptolemaic*. The diagram shows the stationary Earth (E), the orbit of the Sun (S), and the deferent and epicycle of the planet (P). Note that the center of the epicycle lies on the Earth-Sun line. The angle at the planet P in the triangle is a right angle.

Once they have constructed their Ptolemaic model, students are in a position to evaluate Ptolemy's theory. The model matches the observational data both qualitatively and, in the case of their simple solar systems, quantitatively. In particular, the model can reproduce retrograde motion and it automatically makes planets brighter during retrograde because they are closer to Earth at that time. However, there are also some odd features of the model. Retrograde can be synchronized to opposition/conjunction only by linking the motion of each planet to that of the Sun, as described above. No explanation is offered for why planets come in two different types, with different motions for each type. Although the ratio of a planet's epicycle to its deferent is fixed, there is no set scale for relating the size of one planet's orbit to another. Thus, even the order of the planets is not determined in the Ptolemaic system.

4. COPERNICAN MODELING

In the next set of activities, students use the *Copernican System* EJS program (Timberlake 2013) to explore a simplified version of the Copernican theory in which each planet moves uniformly counterclockwise on a circle centered on the Sun (see Note-2). Students discover that the Earth, now treated as a planet that rotates to produce the apparent rotation of the Celestial Sphere, must have an orbital period T_E that is equal to one sidereal year. They find that all other planets are naturally classified as inferior or superior depending on whether their orbit is smaller or larger than Earth's, respectively. The simulations help students discover a formula for the period T_p of a planet's orbit

$$T_p = (T_E^{-1} \pm T_s^{-1})^{-1}, \tag{4}$$

where T_E is Earth's orbital period and T_s is the planet's synodic period (+ for inferior, −for superior).

Students also find that the relative sizes of all planetary orbits can be determined from observations. The ratio of the orbital radius R_I for an inferior planet to the orbital radius R_E of Earth is given by

$$\frac{R_I}{R_E} = \sin\alpha, \tag{5}$$

where α is the planet's maximum elongation as shown in Figure 4.

[2)] Note 2: In the full Copernican system, planets moved on small epicycles which in turn moved uniformly on circular orbits that were not centered on the Sun. These small epicycles effectively reproduced the non-uniform motion of the Ptolemaic system. The theories for Mercury and Venus had additional complications.

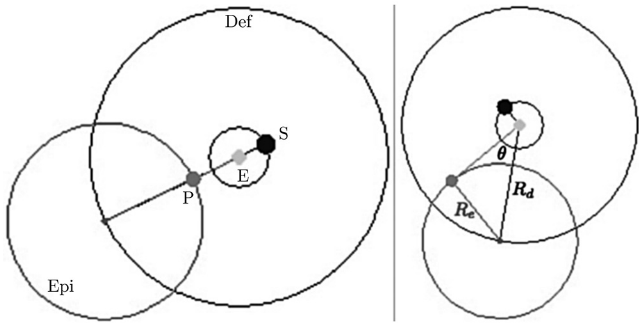

Figure 3. Ptolemaic geometry for a superior planet at opposition (left) and at eastern quadrature (right), modified from *Superior Ptolemaic*. Both diagrams show the stationary Earth (E), the orbit of the Sun (S), and the deferent and epicycle of the planet (P). Note that the line from the center of the epicycle to the planet P is always parallel to the Earth-Sun line. The angle at the planet P in the triangle is a right angle.

Once again the formula is more complicated for superior planets. The geometry at quadrature (Figure 5) shows that

$$\frac{R_E}{R_S} = \sin\theta, \tag{6}$$

where R_S is the radius of the superior planet's orbit and θ is an angle that cannot be measured directly from Earth (it is actually the maximum elongation of Earth as seen from Mars). As in the Ptolemaic system, students can consider the motion from opposition to quadrature to find that

$$\theta = 90^\circ + 360^\circ \frac{t_Q}{T_p} - 360^\circ \frac{t_Q}{T_E} = 90^\circ - 360^\circ \frac{t_Q}{T_s}, \tag{7}$$

where t_Q is the planet's time from opposition to quadrature, T_p is the planet's orbital period, T_E is Earth's orbital period, and T_s is the planet's synodic period. The final equality in Eq. (7) follows from Eq. (4) for superior planets.

After exploring the simplified Copernican model of our solar system, students can develop a Copernican model for their personalized fictitious solar system using the observational data they collected earlier. The most common error in constructing the model is using the incorrect sign in Eq. (4), or simply failing to recognize that the formula for the orbital period is different for inferior and superior planets. Some students also have difficulty

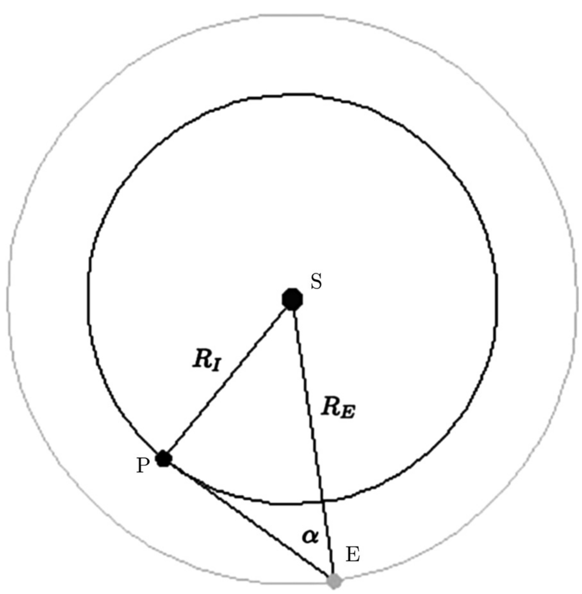

Figure 4. Copernican geometry for an inferior planet at maximum elongation, modified from *Copernican System*. The diagram shows the stationary Sun (S), the orbit of the Earth (E), and the orbit of the planet (P). The angle at P in the triangle is a right angle.

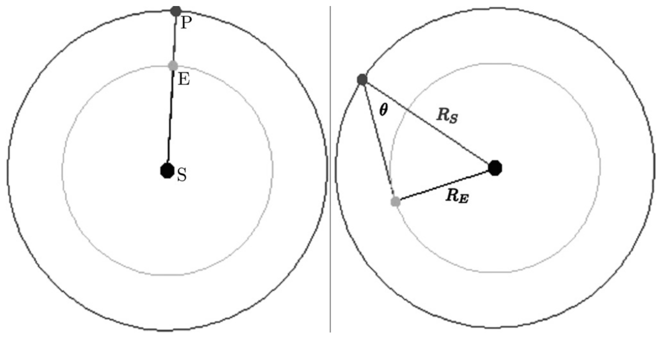

Figure 5. Copernican geometry for a superior planet at opposition (left) and eastern quadrature (right), modified from *Copernican System*. Both diagrams show the stationary Sun (S) and the orbits of Earth (E) and the planet (P). The angle at the Earth in the right diagram is a right angle.

with Eq. (7) because it involves so many quantities and they may get them mixed up. Even students who do all of the calculations correctly may have trouble constructing a properly scaled diagram of their solar system.

Once students have constructed their Copernican model, they can also compare this model to their Ptolemaic model. The Copernican model matches the observational data just as well as (but no better than) the Ptolemaic model, but without any special constraints. The Copernican model provides a natural explanation for the distinction between inferior and superior planets. The Copernican model produces retrograde motion in a natural way when Earth passes, or is passed by, another planet. Retrograde motion is automatically synchronized to conjunction/opposition and also automatically occurs when the planet is closest to Earth (and thus brightest). The sizes of all planetary orbits are fixed relative to Earth's orbit, which provides a unique ordering for the planets. The Copernican model also exhibits a harmony between orbital size and orbital speed: planets closer to the Sun orbit at greater speeds and thus have shorter orbital periods.

It might seem that the Copernican model is superior to the Ptolemaic model in many ways, but historically there were serious problems with the Copernican model. It postulated motions of the Earth that were undetectable and which contradicted the established (Aristotelian) physics, as well as common sense. It also predicted an annual parallax of the stars, which was not observed (Timberlake 2012). For these and other reasons, the Copernican theory was not readily accepted in spite of its many aesthetically pleasing qualities (Martin 1984).

5. PTOLEMY, COPERNICUS, AND TYCHO

The Tychonic theory is essentially a Copernican theory with a stationary Earth. This theory served as a compromise between the Ptolemaic and Copernican systems: it shares many of the aesthetic properties of the Copernican theory, while avoiding the problems of a moving Earth. Students can use the *Ptolemy Copernicus*

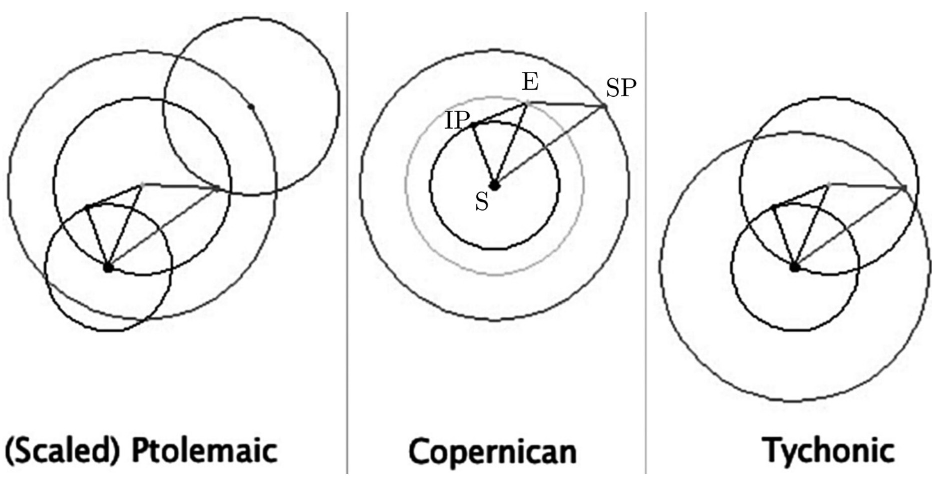

Figure 6. Comparison of simplified versions of the (scaled) Ptolemaic, Copernican, and Tychonic models, modified from *Ptolemy Copernicus Tycho*. All three diagrams show the Earth (E), the Sun (S), one inferior planet (IP), and one superior planet (SP). In the Ptolemaic and Tychonic models the Earth is stationary, while in the Copernican model the Sun is stationary.

Ptolemaic element	Copernican element
Orbit of Sun	Orbit of Earth
Inferior deferent	Orbit of Earth
Inferior epicycle	Orbit of inferior planet
Superior deferent	Orbit of superior planet
Superior epicycle	Orbit of Earth

Table 1. Correspondences between the simplified Ptolemaic and Copernican models

Tycho EJS program (Timberlake 2013) to demonstrate the geometric equivalence of the simplified versions of these three theories (see Figure 6). Note that the Ptolemaic theory has been scaled so that the deferent of the inferior planet, and the epicycle of the superior planet, are the same size as the Sun's orbit. Ptolemy would not have used this scaling, but it is geometrically permitted by his theory.

This program also can help students explore the relationships between the theories. For example, the orbit of an inferior planet in the Tychonic model is identical to the same orbit in the Ptolemaic model. For superior planets, the Tychonic model just swaps the epicycle and deferent from the scaled Ptolemaic model. Once they understand these connections between the Tychonic and Ptolemaic models, students can identify the connections between the Copernican and Ptolemaic models (Brehme 1976). These connections are detailed in Table 1, which lists the elements of the Ptolemaic model and the elements of the Copernican model to which they correspond.

Students can verify that these connections are consistent with the results of their earlier modeling. For example, Eqs. (1) and (5) show that the ratio of an inferior planet's orbital radius to Earth's orbital radius in the Copernican model is equal to the ratio of that planet's epicycle radius to its deferent radius in the Ptolemaic model, as we should expect based on Table 1.

6. CONCLUSION

The activities described in this paper not only allow students to engage in developing and testing scientific models, but they also show students that the same data can lead to very different models that use fundamentally different assumptions. Students gain a deep understanding of the simplified Ptolemaic and Copernican models, including how to construct Ptolemaic and Copernican models using their own observational data. By exploring more than one model of planetary motion, students have the opportunity to evaluate the theories based on empirical adequacy, consistency with other accepted theories, and even aesthetic criteria. In short, students are given the opportunity to engage in the process of doing science.

If these activities and projects are followed by a discussion of how Galileo, Newton and others introduced new ideas about the physics of motion, then students can learn about how developments in one area of science can lead to the re-evaluation of theories in other areas. In the 16th century the Copernican model conflicted with the accepted (Aristotelian) physics and was rejected in favor of the Ptolemaic or Tychonic models. By the 18th century Newtonian physics became the dominant physical theory, and the Copernican model became accepted, in spite of the fact that it predicted an as-yet-unobserved annual stellar parallax, because it fit much better with Newtonian ideas about motion than did the Tychonic theory. Studying this episode in the history of astronomy can give students significant insight into how scientific theories are evaluated and how those evaluations change over time, thus helping students gain a better understanding of the nature of science.

References

Brehme, R. W. 1976, "New look at the Ptolemaic system," *American Journal of Physics*, 44, 506.

Crowe, M. J. 1990, *Theories of the World from Antiquity to the Copernican Revolution*, New York: Dover.

Duncan, D. K., and Arthurs, L. 2012, "Improving Student Attitudes about Learning Science and Student Scientific Reasoning Skills," *Astronomy Education Review*, 11, 010102–1.

Esquembre, F. 2013. Easy Java Simulations computer program, http://fem.um.es/Ejs/. Accessed 13 January, 2013.

Etikina, E., Warren, A., and Gentile, M. 2006, "The Role of Models in Physics Instruction," *The Physics Teacher*, 44, 34.

Kuhn, T. S. 1985, *The Copernican Revolution*, Cambridge, MA: Harvard University Press.

Linton, C. M. 2004, *From Eudoxus to Einstein: A History of Mathematical Astronomy* Cambridge, UK: Cambridge University Press.

Martin, D. R. 1984, "Status of the Copernican theory before Kepler, Galileo, and Newton," *American Journal of Physics*. 52, 982.

Matthews, M. R. 1994, *Science Teaching: The Role of History and Philosophy of Science*, New York: Routledge.

Open Source Physics 2013. Open Source Physics collection, http://www.compadre.org/osp/. Accessed 13 January, 2013.

Stellarium 2013. Stellarium computer program, http://www.stellarium.org/. Accessed 13 January, 2013.

Timberlake, T. 2012, "Seeing Earth's Orbit in the Stars: Parallax and Aberration," http://arxiv.org/abs/1208.2061.

Timberlake, T. 2013, Modeling the History of Astronomy shared folder, http://www.compadre.org/OSP/filingcabinet/share.cfm?UID=12250&FID=33000&code=A816D1F75A. Accessed 13 January, 2013.

Wittman, D. 2009, "Shaping Attitudes Toward Science in an Introductory Astronomy Course," *The Physics Teacher*, 47, 591.