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Eratosthenes 2009/2010: An Old Experiment in Modern Times

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Abstract

We describe the projects "Eratosthenes 2009" and "Eratosthenes 2010 America," carried out during the International Year of Astronomy (2009) in Argentina and with almost all the countries in North and South America during 2010. More than 15 000 students at more than 200 schools each year determined the radius of the Earth using the method that Eratosthenes employed more than 2000 years ago. The result obtained was 6290 km in 2009 and 6375 km in 2010, in good agreement with tabulated values of 6371 km [McCarthy and Petit, 2004, "IERS Conventions (2003)," IERS Technical Note No. 32, Frankfurt: Bundesamts für Kartographie und Geodäsie]. These projects involve history, mathematics, and astronomy to create an exciting activity with accurate scientific results.

1. INTRODUCTION

Eratosthenes—the famous mathematician, Greek astronomer, geographer, and poet—lived in Alexandria, Egypt, during the third century B.C. He obtained the first accurate determination of the circumference of the Earth about 2300 years ago.

Eratosthenes knew that on one day, at noon, in Siena (now Aswan), a city located south of Alexandria, sunlight fell perfectly vertically in a deep well. This observation meant that the Sun was directly over the

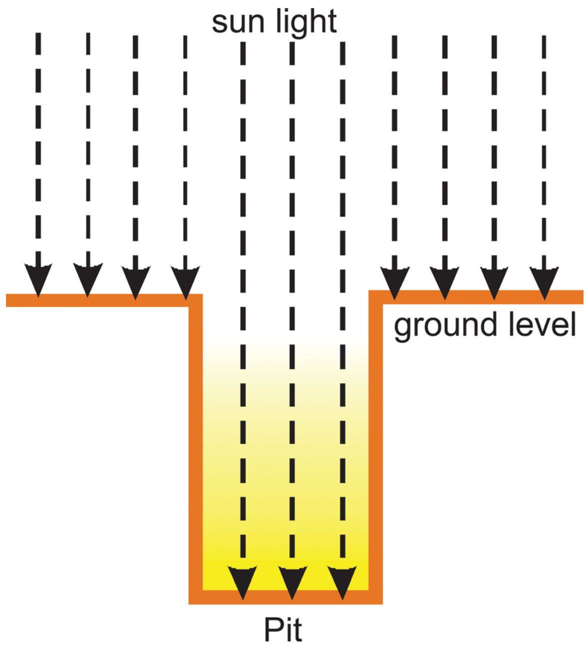

Figure 1. The Sun's rays fall into the vertical shaft located in Siena, when the Sun is exactly over this city (on June 21 at noon). At that moment, the walls do not project any shadow.

city of Siena, as shown in Figure 1. Eratosthenes also knew that while this was taking place in Siena, the same thing was not happening in Alexandria. (See Figure 2; note that in both figures the Sun's rays are parallel to each other.)

Eratosthenes used a shadow like this to calculate the circumference of the Earth. When the Sun was directly over Siena (at noon on June 21), he measured the shadow of an object in Alexandria. Knowing the length of the object, its shadow's length, and the distance between Alexandria and Siena, he was able to estimate the circumference. The value obtained was very similar to the current value.

This is a very instructive, ingenious, and simple experiment that illustrates the methods used in science to describe natural phenomena. Moreover, it provides a concrete answer to a question that has had an enormous relevance in the past.

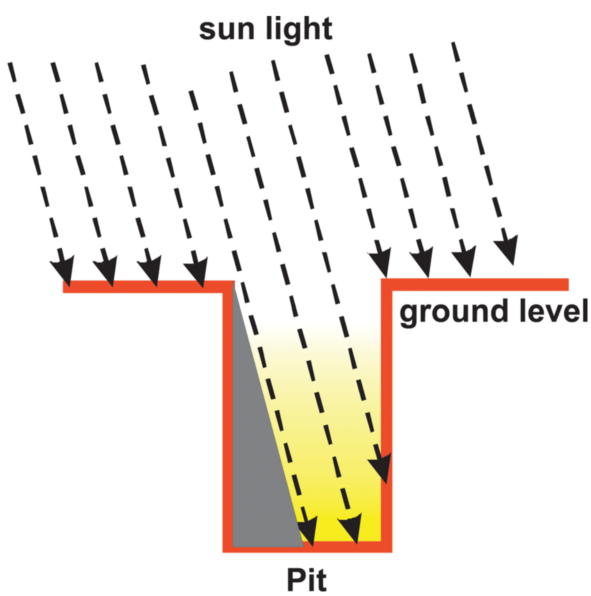

Figure 2. At the same time, when in Siena sunlight enters the pit as in Figure 1, in Alexandria its rays enter at an angle to the vertical. Here the Sun is partially blocked by the pit walls, and there is a shadow (which is exaggerated in the drawing).

2. OBJECTIVES

The main goal of these projects was the experimental determination of the value of the radius of the Earth by high-school students, using the method devised by Eratosthenes. Since the World Year of Physics 2005, these projects have been carried out in Argentina under the supervision of several institutions, based on similar activities in France (Anonymous 2009, 2010) and the United States (WYP 2005). Last year, in commemoration of the International Year of Astronomy, the experience involved a large number of students and schools, primarily in Argentina.

The projects have several purposes. Among them, it is expected that students learn how to describe the geometry of the rays of sunlight on the surface of the Earth at different latitudes, to determine real noon at the place where they live from observations of the apparent motion of the Sun, and to apply simple trigonometric relations to obtain the radius of the Earth from their measurements. Moreover, since the experiment involves schools at different geographic locations, the students are part of a collective project, in which, with input from various groups not only in the same country, they can achieve a specific goal.

3. THE EXPERIMENT OF ERATOSTHENES TO DETERMINE THE CIRCUMFERENCE OF EARTH

In Figure 3, Siena is represented by S and Alexandria by A, two points on the surface of the Earth. The arc length between S and A is d, and the angle of this arc is θ. The radius of the Earth is R. Suppose that the Sun's rays reach the Earth from a given direction (i.e., they are approximately parallel to each other). At the moment, the Sun is just above the city of Siena, the rays arrive perpendicular to the surface in that city, and therefore have the same direction as the radius of the Earth that connects Siena with the center of the terrestrial sphere.

If we analyze the experiment of Eratosthenes using concepts of trigonometry (which had not yet been discovered in his time), we can describe the relationship between the height of a tower, the length of its shadow, and the angle θ (Figure 4) as

$$\tan(\theta) = \text{length of the shadow/height of the tower.} \tag{1}$$

Knowing the tangent of the angle, $\tan(\theta)$, and the fact that θ is smaller than 90°, it is possible to determine θ. The proportion of the total circumference of the Earth (C), which represents the arc length (d) joining S (the city of Siena) and A (the city of Alexandria) on the Earth's surface, is equal to the proportion that represents the angle θ with respect to the angle of 360°. Therefore,

$$d/C = \theta/360, \tag{2}$$

and the circumference of the Earth can be written in terms of measurable quantities

$$C = 360d/\theta. \tag{3}$$

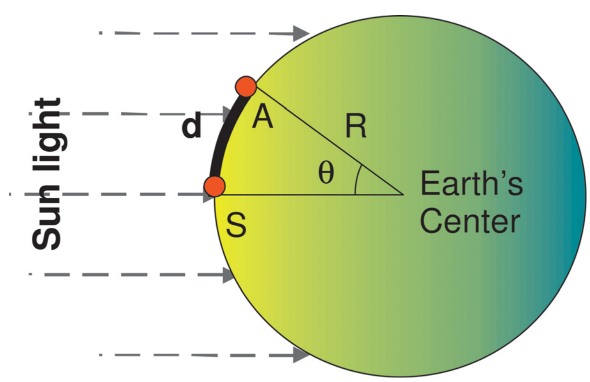

Figure 3. Relationship between the angle at the center of the Earth and the arc on the surface.

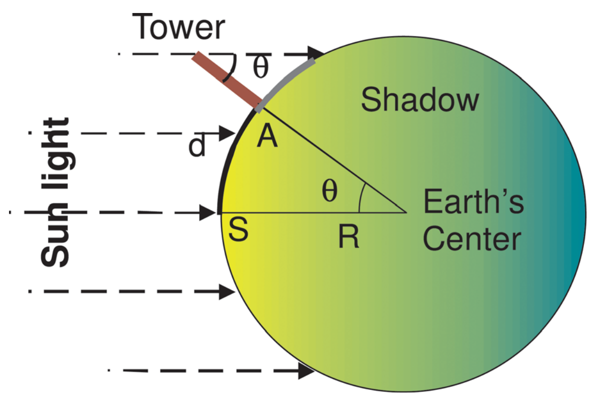

Figure 4. Geometry of our Eratosthenes experiment: Measure the height of a tower and length of its shadow at noon on June 21 in Alexandria. Then calculate the angle formed by the Sun's rays to the vertical in this city. This angle coincides with the north–south angular distance between the cities of Siena and Alexandria.

4. ERATOSTHENES ON JUNE 21 UNDER OR OVER THE TROPIC OF CANCER

Eratosthenes knew a place where the Sun fell exactly vertically at noon. If this is not the case, the radius of the Earth can still be determined from the measurements of the direction of incidence of sunlight at two different locations, as shown in Figures 5 and 6. Let us consider two schools located approximately on the same terrestrial meridian, but at different latitudes, both north and south of the Tropic of Cancer.

Schools, located at points A and B are separated by a distance *d*, in the north–south direction. Students from each school measured the angle formed by the Sun's rays to the vertical at noon at the place where each school is located. We call these angles θ_A and θ_B.

The students from the school located at A measured the length of their stake and the shadow projected by it at noon, the same day as did students from the school located at B. From this measurement, it is possible to calculate the tangent of the angle formed by the Sun's rays with the vertical, θ_A

$$\tan(\theta_A) = \text{length of the shadow}/\text{length of the stake}. \tag{4}$$

The students from the school located at B did the same measurements and obtained θ_B.

The angle that underlies the arc joining the points A and B is the difference between θ_A and θ_B. Therefore, we can use the formula (3), replacing the circumference, *C* by $2\pi R$, and taking into account that instead of the angle θ we must use the difference between θ_B and θ_A. That is,

$$2\pi R = 360d/(\theta_B - \theta_A). \tag{5}$$

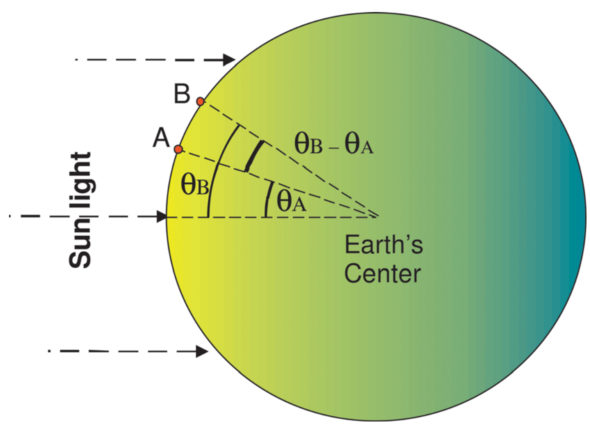

Figure 5. Geometry to measure the radius of the Earth using data from two schools working together in conducting the experiment.

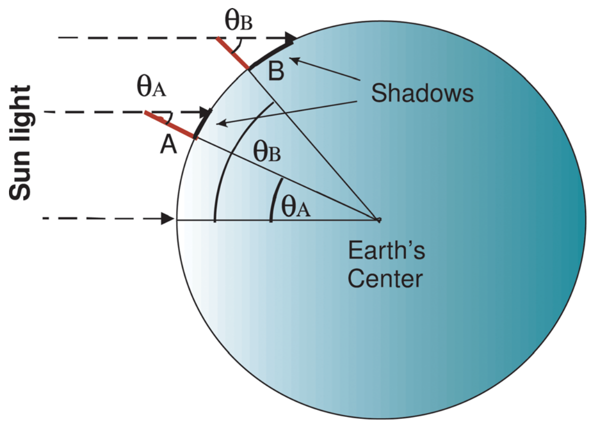

Figure 6. Relationship between the direction of the Sun's rays, the stakes, and the two angles, $\theta_{\rm A}$ and $\theta_{\rm B}$.

From this equation, the radius of the Earth can be written in terms of d and the angles obtained by each school

$$R = 180d/\pi(\theta_{\rm B} - \theta_{\rm A}). \tag{6}$$

There are several issues to be taken into account when performing the measurements: ideally, both schools should measure the angles on the same day. In order to avoid large errors in the determination of R, the distance d between schools should be as large as possible. Finally, the measurement of the angle must be performed at the solar noon, that is, when the length of the shadow is at a minimum. In general, this will not take place when the local time is 12.00 noon.

The north–south distance between each pair of schools was obtained using one of the tools available on the Internet, based on the calculation of the distance of each school to the equator along the meridian of the site. As both schools are almost on the same meridian, the distance between them must be approximately equal to the difference between the distances from each school to the equator, determined by the latitude of the cities where the measurements were done (see for example: http://www.tutiempo.net/p/distancias/calcular_distancias.html).

When the two schools are on different sides of the Tropic of Cancer, Eq. 6 remains valid for the calculation of the radius, but what should be kept in mind is the convention that the angles involved have opposite signs. Figure 7 illustrates the situation.

The difference of angles (θN $-$ θS) needed to calculate the radius, as in the previous section, in this case involves a negative value (θN $<$ 0); then the difference becomes the sum of the absolute values of both angles.

5. METHODOLOGY

Once the schools registered their geographical coordinates through the web link http://df.uba.ar (Eratosthenes icon), we proposed an optimum distribution of pairs of schools as follows. We used three integer linear programming models, taking as parameters the list of participant schools, and the longitude and latitude of each one. The aim of the first model was to obtain a linkage that maximizes the minimum north–south distance between partner schools, subject to the restriction that all schools have at least two and at most six couples. Then, imposing the optimum of the first model as a lower bound for the east–west distances for couples, we applied a second model in order to achieve a coupling that minimizes the maximum east–west distance. Finally, the objective of the third model was to minimize the number of coupling links, imposing the optimum of the first model as a lower bound for the north–south distances for couples, the optimum of the second model as an upper bound for the east–west distances for couples, and the restriction that all schools have at least two and at most six couples. These models were solved with ZIMPL (Koch 2009) + SCIP (Achterberg *et al*. 2009), free software developed by Tobias Achterberg, Timo Berthold, Thorsten Koch, and Kati Wolter at the Zuse Institute Berlin.

Once the distribution of pairs of schools was established, we published the list on the homepage of the projects. Then, both schools of each pair agreed on the measurement day (around June 21, depending on the school's calendar or the weather report), the experimental method, the noon determination, the uncertainty criteria, the

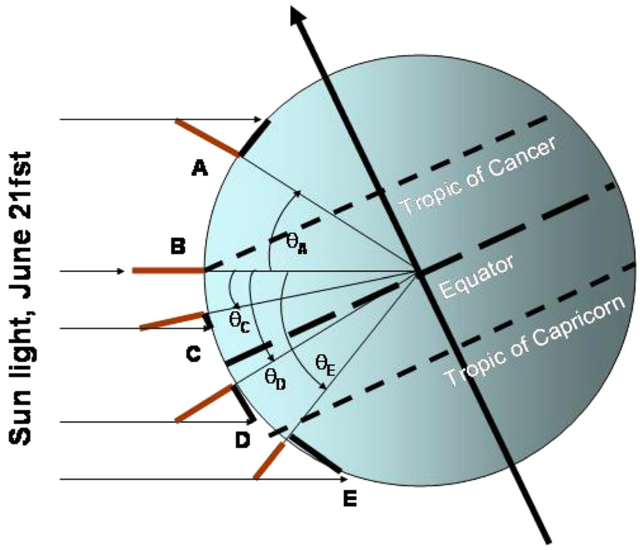

Figure 7. Relationship between the direction of the Sun's rays on June 21, the stakes, and the angles north and south of the Tropic of Cancer.

students' job, and the possible social interaction between schools. Particularly we emphasized the importance of the experimental determination of the solar noon over known calculated values.

Finally, each school uploaded its data through the web so we were able to do a statistical analysis and obtain a single value of the Earth radius. In Figures 8 and 9, we present a group of students who performed the experiment.

6. ERATOSTHENES IN ARGENTINA: INTERNATIONAL YEAR OF ASTRONOMY (IYA) 2009

The measurements were made between 18 and 24 June 2009. The project involved 258 schools from 18 provinces in Argentina, 9 from Uruguay, and 1 from Chile—over 15 000 students in total. Using the optimization methods described above, 207 pairs of schools were formed. One hundred fifty seven of them made their measurements simultaneously, while 50 pairs were in fact schools that matched their measurements with a hypothetical one on the Tropic of Cancer, because their partners left the project.

The values obtained for each pair of schools for the terrestrial radius R varied between 2000 and 9000 km. With the results, a histogram was made that was fitted with a Gaussian distribution to determine the standard deviation, σ, and the average value R. The statistical uncertainty, ξ, was calculated in the usual way, as

Figure 8. Students of Colegio Central Universitario, San Juan.

Figure 9. Students of Escuela del Magisterio, National University of Cuyo, Mendoza.

$\xi = \sigma/\sqrt{N-1}$, where N is the number of reported values. The result is $R = 6320 \pm 20$ km. The reported error is only statistical, as we have not required the schools to provide the errors in their measurements.

The main sources of errors in the determination of the lengths of the gnomon and the shadow may arise from the "ruler" resolution, thickness of the gnomon, discrimination between umbra and penumbra at the end of the shadow, bad orientation of the gnomon, and surface where the shadow is projected. Another source of error is the time of the day at which the measurements were preformed. Typically, measuring one hour ahead or behind the solar noon can result in more than 5% error.

Given the complexity of the determination of the nonstatistical error in each case, we will provide in the future a tutorial for teachers, so that they are informed on estimation of different sources of errors, so that we can evaluate the final uncertainty.

Indeed, in most cases, the schools did not discuss this issue. This is understandable considering that it has a level of complexity too high for discussion in class. However, according to the tabulated values (McCarthy and Petit 2004), the Earth's radius $R = 6378$ km, so the outcome of the experiment was satisfactory.

The Argentinean Project Eratosthenes 2009 on June 21st participated in the festival dedicated to Eratosthenes, organized by the New Library of Alexandria (Egypt) (Bibliotheca Alexandrina 2009), in a video conference in which students and teachers in Alexandria, Aswan (Egypt), Virginia (United States), and Buenos Aires (Argentina) commented on their results and methods of making the measurements (see Figures 10–13).

Taking into account the comments and reports received from teachers and students, large number of participants, and the results obtained, we consider that this has been a highly successful activity.

Figure 10. Video Conference in Alexandria, Egypt, June 21, 2009.

Figure 11. Students at a school of Base Esperanza, Argentinean Antarctica, 63:23:42S; 56:59:46W.

7. ERATOSTHENES AMERICA: 2010

The proposal during 2010 was to open the project to all American high schools. For this purpose, we configured a web network of scientists in different countries (university professors, researchers from different institutes, and members of professional associations) in order to help the schools to perform the experiment. The Universidad Tecnológica Nacional, Regional Mendoza, Argentina, and Asociación Física Argentina, Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Argentina, were the organizing committee. The Asociación Argentina de Astronomía; Observatorio Naval Argentino; Olimpíada Brasileira de Astronomia e Astronáutica; Universidade Federal de Mato Grosso do Sul; Olimpiáda de Astronomía en México, Instituto Nacional de Astrofísica, Óptica y Electrónica; AIA 2010 Paraguay; Observatorio Consejo de Educación Secundaria de Uruguay; Centro de Investigaciones de Astronomía "Francisco J. Duarte" de Venezuela; and Network for Astronomy School Education were institutions that linked the schools. The Asociación Física Argentina (Filial Buenos Aires) and Ministerio de Ciencia, Tecnología e Innovación Productiva, supported the project.

Two hundred twenty six schools in North and South America were part of the experiment in 2010. The number of students involved in the activity exceeded 15 000. The measurements were scheduled between 18 and 24 June 2010. The pairs of schools which contributed to the joint measurement were 174, according to the optimization methods based on the geographical coordinates of all the participants and the day of measurement.

Figure 12. Students at the school of Hidalgo, Mexico.

Figure 13. Students at the school of Sao Paulo, Brazil.

The values obtained for each pair of schools for the terrestrial radius R varied between 4000 and 8000 km. A histogram of the results was made, and the results were fitted to a Gaussian distribution. As described above, the average value and the statistical uncertainty were $R = 6375 \pm 25$ km. As we pointed out before, the recorded error is only statistical, as we have not requested the schools to inform us of the errors of their measurements. According to the tabulated values (McCarthy and Petit 2004), the Earth's radius $R = 6371$ km, so the outcome of the experiment was excellent.

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