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Distance to the Center of the Milky Way Galaxy: An Experiment for Intermediate-Level Students Using Research Data and Professional Analysis Tools

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Abstract

The most recent data on stars orbiting Sagittarius A*, the black hole at the center of the Milky Way galaxy, have been used in an experiment undertaken by second-year university students to determine the distance $R\theta$ to the center of the galaxy. Students applied each of Kepler's 17th-century laws, in turn, to 21st-century data. The experiment, pedagogic considerations, and student learning are described.

1. INTRODUCTION

The laboratory component of an intermediate-level astronomy subject at Monash University in Australia was subjected to a major overhaul in 2007. The instructors' goals were to engage students through the use of contemporary data, to equip them with analysis skills that would be useful in later years, and, where possible, to have them handle instruments to collect their own data.

Although much current research is beyond the scope of understanding of intermediate-level students, the conceptual pathway travelled by Eisenhauer et al. (2003) in determining the most direct measurement of the distance to the center of the Milky Way galaxy could be followed and understood by these students. Determination by a geometric approach of the fundamental parameter R_θ, the distance to the center of the galaxy, is a powerful learning experience for students.

The team around Reinhard Genzel at the Max-Planck-Institut für extraterrestrische Physik (MPE) (Note 1) in Germany imaged the Galactic Center once each year from 1991 to 2002 using the European Space Organisation’s 3.6 m New Technology Telescope (NTT) telescope in La Silla, Chile, with the specially designed Sharp I Infrared camera. In 2002, an improved Near-Infrared Imager and Spectrograph (CONICA) and adaptive optics combination (Nasmyth Adaptive Optics System [NAOS]) were installed on one of the foci of Yepun, the fourth telescope of the ESO 8 meter-class Very Large Telescope (VLT) array, providing higher-resolution images. For one of the stars orbiting Sgr A*, 12 data points (12 years) represented an almost complete orbit. Kepler’s laws yielded the orbit’s dimensions, and simple geometry then yielded the distance to Sgr A* (Schödel et al. 2003; Eisenhauer et al. 2003).

2. THE DATA

One of the authors (MTF) sought permission from Frank Eisenhauer at MPE to use the original data. He and his colleague Stefan Gillessen had misgivings about the accessibility of crowded field data for student use and generated a clean data set based on the original, incorporating new data from 2004 to 2006; that is, they provided 15 years of data.

The Flexible Image Transport System (FITS) cube contains a representation of the data from the Galactic Center observations analyzed by Eisenhauer et al. (2003) and is supplemented by data from 2004, 2005, and 2006: 15 images representing observations taken one year apart, starting at 1992.5 and ending at 2006.5. In each image, the positions of the stars *S2*, *S1*, *S8*, *S12*, *S13*, and *S14* were represented by a typical adaptive optics point spread function (PSF). Each 200 × 200-pixel image in the cube represents a 1.2 × 1.2-arcsecond field of view. Some of the images are shown in Figure 1.

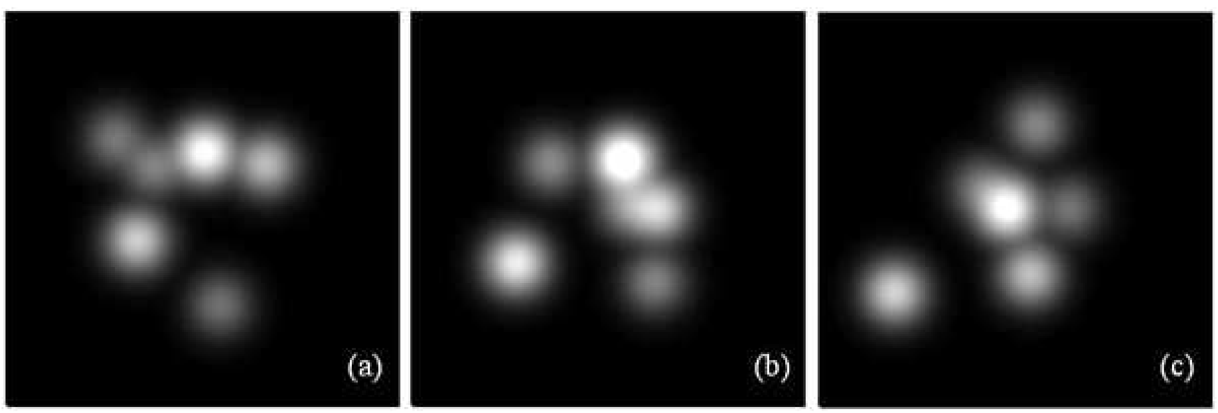

Figure 1. Example of data from the FITS cube. (a) 1992.5 (b) 1996.5 (c) 2002.5

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3. THE EXPERIMENT

Although students did not collect their own data, the "distance to the galactic center" experiment allowed them to develop familiarity with professional astronomy analysis tools and not only to replicate the result of Eisenhauer et al. (2003) but also to use more recent data.

There are a number of useful constants in astronomy: the luminosity of the Sun **L**υ, the mass of the Earth $\mathbf{M}_{\mathbf{Earth}}$, and so forth. There are several distance constants: the average Sun–Earth distance, **1 AU**, the **parsec**, the **light-year**. Another is $\mathbf{R}_{\theta}$, the distance from the Sun to the center of the Milky Way galaxy. $\mathbf{R}_{\theta}$ is essential to the calibration of brightness of standard candles such as RR Lyrae stars and Cepheids and also permits accurate mapping of the galaxy with 21 cm radiation and CO regions.

A discussion of the importance of R_{θ} as a fundamental unit in astronomy—much like L_{Sol}, M_{Earth}, and the light-year—begins the laboratory script. In a later experiment, students use R_{θ} to determine the rotation curve of the Milky way from 21 cm radio observations and to construct a galaxy map so that their determination of R_{θ} fits into a coherent program.

Generally, students were guided rather than directed in their analysis of the data. Because they were second-year students, few problems emerged, although treatment (or lack of treatment) of uncertainties emerged as an issue that needed attention.

The script provided students with background information on the data set they would use, and some "preliminary questions" reminded them of Kepler’s laws. Students were introduced to the Linux operating system and the Starlink software, and to FITS images and the use of GAIA (Graphical Astronomy and Image Analysis tool). They were guided through basic steps with directions such as:

> Use Kepler’s Second Law to calculate the orbital period of *S2* about the black hole. *How? Try to come up with a graphical technique*. **Discuss with your demonstrator** before venturing off into the unknown. Some useful properties of ellipses and trigonometry are shown at the end of this script. Also consider whether your technique will be accurate and whether it will have systematic errors.

They examined the image data set and selected the most suitable star for analysis. They determined orbital parameters and then the distance to Sgr A* at the center of the Milky Way. Students achieved good results quite comfortably in the three-hour laboratory session available.

3.1 Preliminary Questions

Before the laboratory session, students were required to attempt some relatively basic questions to reacquaint themselves with the ideas behind Kepler’s laws (Kepler 1609; Kepler, Ptolemaeus, & Fludd 1619).

Students were required to:

- Draw an ellipse, showing the semimajor and semiminor axes
- Determine the eccentricity of their arbitrary ellipse and hence to locate the foci
- Draw on the ellipse a graphical representation of Kepler’s Second Law (equal areas swept out in equal times)
- Consider Kepler’s original Third Law $a^3 = k P^2$ and Newton’s more general expression

$$\left(\frac{P}{2\pi}\right)^2 = \frac{a^3}{G\,(M+m)}$$

and hence to determine how Kepler's constant of proportionality *k* would change if the central mass *M* were varied

- Consider whether Kepler's laws would be applicable if an orbit was inclined with respect to our line of sight

3.2 Introduction to the Professional Tools

This was the first computational experiment undertaken by our second-year astronomy students. The first-year astronomy subjects cover a broad syllabus, including positional astronomy, the Solar System, stars, galaxies, cosmology, and life in the universe. In second year, students encounter a more rigorous and focused syllabus, properly interacting with the professional tools of astronomy. This can be very intimidating for some students with weaker computer skills, and so this section of the laboratory script led students in a recipe book directive style in the use of Linux, GAIA, and FITS files. The later sections of the script were much less directive and more open-ended, requiring independent thought.

Students were led step by step through logging into a Linux operating system and instructed in the use of a terminal. The command-line was not used extensively (only to load the software to be used).

The most commonly used software in the astronomy community is IRAF (Image Reduction and Analysis Facility). Because of the complexity and quirks of IRAF, we opted for the more user-friendly software GAIA for this introduction to computational astronomy.

GAIA was used to look at the data in a FITS cube. This would also likely be the first time that students had encountered a FITS file. That it was a cube and not a single image did not seem to be a particular challenge to students.

3.3 The Analysis

3.3.1 Choosing the Best Star for Orbital Analysis

After the familiarization tasks to learn about FITS files and GAIA, the students started to look at the data themselves. The first issue they had to tackle was determining which of the stars in the field of view had the most useful orbit for analysis. Most of the stars in the FITS cube complete less than half an orbit; some even seem to travel in a straight line. One of the stars, *S2*, travelled almost a complete orbit in the 15 years. The choice of *S2* as the most suitable star for analysis may seem obvious to the more experienced astronomer, but it was not to the students. They took not insignificant time to consult each other and make a decision. Their discussions were useful and on topic, which is quite rare in science laboratories (von Aufschnaiter & von Aufschnaiter 2007; Gunstone et al. 1996).

3.3.2 Determining the Location of Star S2 *in Each Frame*

To generate a plot of the location of *S2* with respect to Sgr A* over the 15-year interval, students needed to determine the position of the star *S2* within each frame. They were required to determine the *x-y* pixel values of *S2* but were given no guidance as to how to achieve this. An experienced astronomer would notice the near-perfect symmetry of the PSF and would locate the center of the star within the brightest pixel. The intensity corresponding to each pixel value follows the cursor, so students quickly realized that they needed to optimize the intensity value, and the pixel at which that was achieved would give the best estimate of the star's location. They "discovered" the brightest pixel method for themselves quite quickly.

Conversion of pixel locations to arcseconds and plotting the location of the star were straightforward for all the students.

3.3.3 Application of Kepler's First Law to the Orbit of Star S2

Students were required to fit an ellipse, representing the use of Kepler's First Law (all planets move in elliptical paths with the Sun at one focus), to their plot of *S2*'s location over time. Half of the students printed out their plot and sketched an ellipse by hand; the others used the ellipse tool in Excel to draw, and manipulate into position, an accurate ellipse. Choice of method was primarily determined by each student's confidence in using computers. An example of one student's data and fitted ellipse is shown in Figure 2.

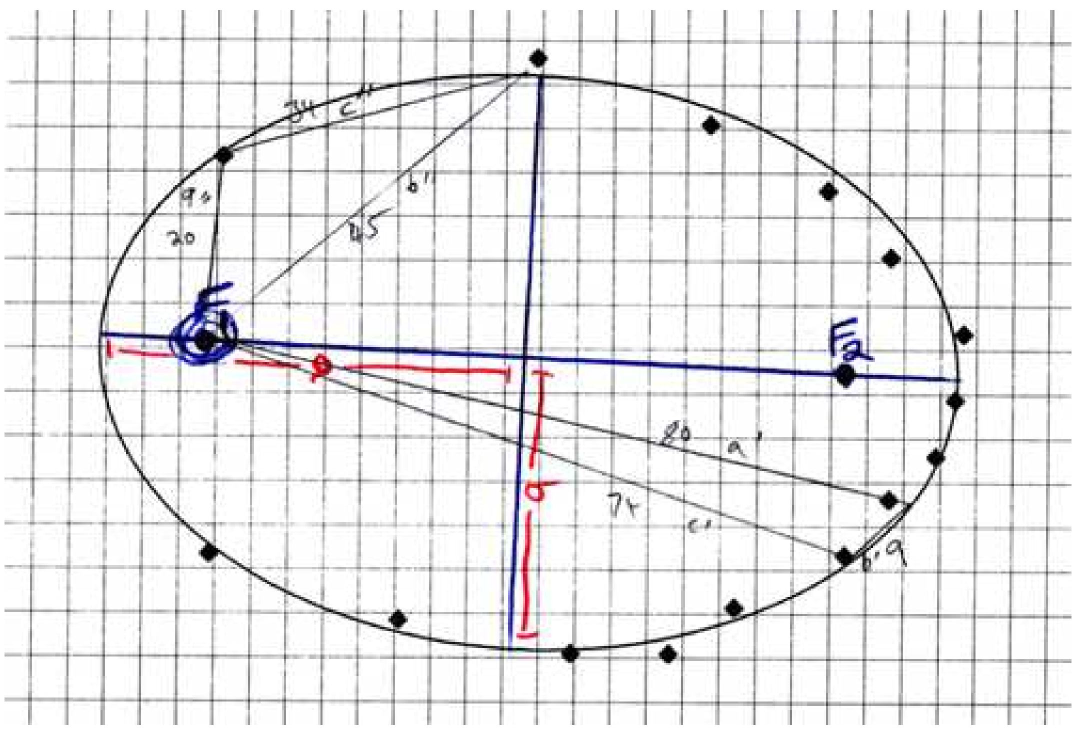

Figure 2. Student David Palamara's plot of location data, with ellipse. Hand-drawn markings are semimajor and semiminor axes, foci, and use of Heron's formula to determine areal velocity.

From the ellipse, the semimajor and semiminor axes, the eccentricity, and the location of the foci were determined. As usual, the laboratory script left the method open to the student. The two most common approaches taken were to print out the plot and measure the axes in millimeters after figuring out a millimeter-to-arcsecond conversion factor, and measuring directly using Excel.

3.3.4 Application of Kepler's Second Law to Determine the Orbital Period of Star S2

The next step proved the most challenging. Using their plot, students needed to apply Kepler's Second Law: Equal areas are swept out in equal times. The only realistic way to do this, aside from writing a program to calculate it or to count squares on graph paper, was to determine the areal velocity (area per unit time) using Heron's formula (Heron of Alexandria A.D. 60):

The area *A* of any given triangle with sides of length *a*, *b*, and *c* is given by

$$A = \sqrt{s \times (s-a) \times (s-b) \times (s-c)}$$

where the semiperimeter of the triangle is

$$s = \frac{a+b+c}{2}.$$

Students used the lengths of each side of a triangle formed from straight lines linking the foci and each of two consecutive observations, and the two observations (as shown in Figure 2). Students all realized that they needed the area of two triangles relating to two sets of consecutive observations.

About half of the students managed to arrive at this method on their own, and the rest needed significant prompting from the instructors.

3.3.5 Application of Kepler's Third Law to Determine the Distance to the Galactic Center

Although determination of the mass of the central black hole was an essential part of the original study, the three-hour time constraint for the laboratory class did not allow its exploration in this experiment. Using Kepler's Third Law and given $M_{\mathrm{Sgr\ A*}}$ = 3.3 × 106 $\mathrm{M}_{\mathrm{Sol}}$ (Shödel et al. 2003), students calculated the semimajor axis of the orbit of star *S2* in astronomical units (AU).

Students could then use simple trigonometry to calculate the distance to the center of the galaxy in parsecs, and the distance of closest approach (tangentially) of star *S2* to the black hole. Although there is no mention in the laboratory script, staff prompted students to consider the effect of inclination because the orbit is unlikely to be perpendicular to our line of sight. Some students had realized this beforehand. In fact, Kepler's laws apply at any inclination because each orbital parameter is scaled by the same factor.

However, inclination of the orbit of star *S2* does affect the determination of its closest approach to the central black hole.

Finally, students were directed toward the literature, using the NASA Astrophysics Data System (ADS), to compare their own estimate of the distance to the center of the galaxy with that determined by Eisenhauer et al. (2003). Students typically determined a value for R_{θ} around 8.3 kpc, with a standard deviation of 0.5 kpc for the 35 students in the group. The researchers' best current estimate for R_{θ} is 7.94 ± 0.42 kpc; students attributed their own overestimations to their poor fitting of an ellipse (not eccentric enough) and/or to the lack of data in the fastest area of star *S2*'s orbit.

4. STUDENTS' LEARNING

Some components of the laboratory script were specifically designed to demand inquiry and decision-making of the student. Here we discuss students' responses to parts of the analysis where they were not given specific instructions or direction.

The laboratory script asked questions such as:

- Which star has the orbit that should yield the useful data? Discuss with your demonstrator and note your discussion in your log book.
- For each image, use the *zoom* panel and the *Value* to pinpoint the location of your selected star, *S2*. How? Discuss with your demonstrator and note your decision in your log book.
- Which of Kepler's laws are you using?
- Which focus corresponds to the location of the black hole? Justify your answer.
- Use Kepler's Second Law to calculate the orbital period of *S2* about the black hole. *How? Try to come up with a graphical technique.* **Discuss with your demonstrator** before venturing off into the unknown. Some useful properties of ellipses and trigonometry are shown at the end of this script. Also consider whether your technique will be accurate and whether it will have systematic errors.
- Once you have determined the period of *S2*'s orbit, consider which of Kepler's laws can be used to calculate the semimajor axis length in astronomical units.
- Calculate the semimajor axis in AU. Together with the dimensions of the semimajor axis in arcseconds, you can use simple trigonometry to determine the distance to *S2* and hence to the galactic center given the estimated mass of the black hole, 2.6 million solar masses.
- In its orbit, how close did *S2* come to the black hole?

Students worked in groups of two or three and responded to the script positively, with a lot of talk among group members as they grappled with each stage of the analysis. They were required to consciously relate theory to the data manipulation and analysis in which they were engaged, and they showed significant intellectual engagement.

Students selected which of Kepler's laws to apply as they followed the script, which guided rather than directed them. They judged which star would yield the most useful analysis, they discovered the brightest pixel method to locate a star, and they converted from pixels to arcseconds, and later to astronomical units, without specific instructions and without difficulty.

The open-ended laboratory script, with a lack of direction regarding the method to be used, led to some interesting improvements and exploration of issues by some of the groups of students. For example,

- One group "invented" a technique to determine the position of star *S2* to subpixel accuracy. Although the center of the star is located within the brightest pixel, giving a pixel location of ($x \pm 1$, $y \pm 1$), this was improved by calculating the subpixel peak position in a 3×3 matrix centered on the brightest pixel.
- Another group was concerned that when *S2* was close to the position of another star, the other star's contribution to the values of the pixels in *S2* would influence their results; they recognized the "crowded field" problem. This, of course, was the very problem foreseen by authors SG and FE and the reason that they had provided simulated data, without crowded fields, for students to analyze.
- Some of the more advanced students recognized that Heron's formula for the area of a triangle would underestimate the area of the sector of the ellipse swept out, and they factored that into their uncertainty estimates.

One group of 12 students completed survey sheets, providing information on their experience of the experiment. Figure 3 summarizes their responses to three questions. Clearly, students found the experiment enjoyable and intellectually challenging and felt that their experience had been realistic rather than contrived.

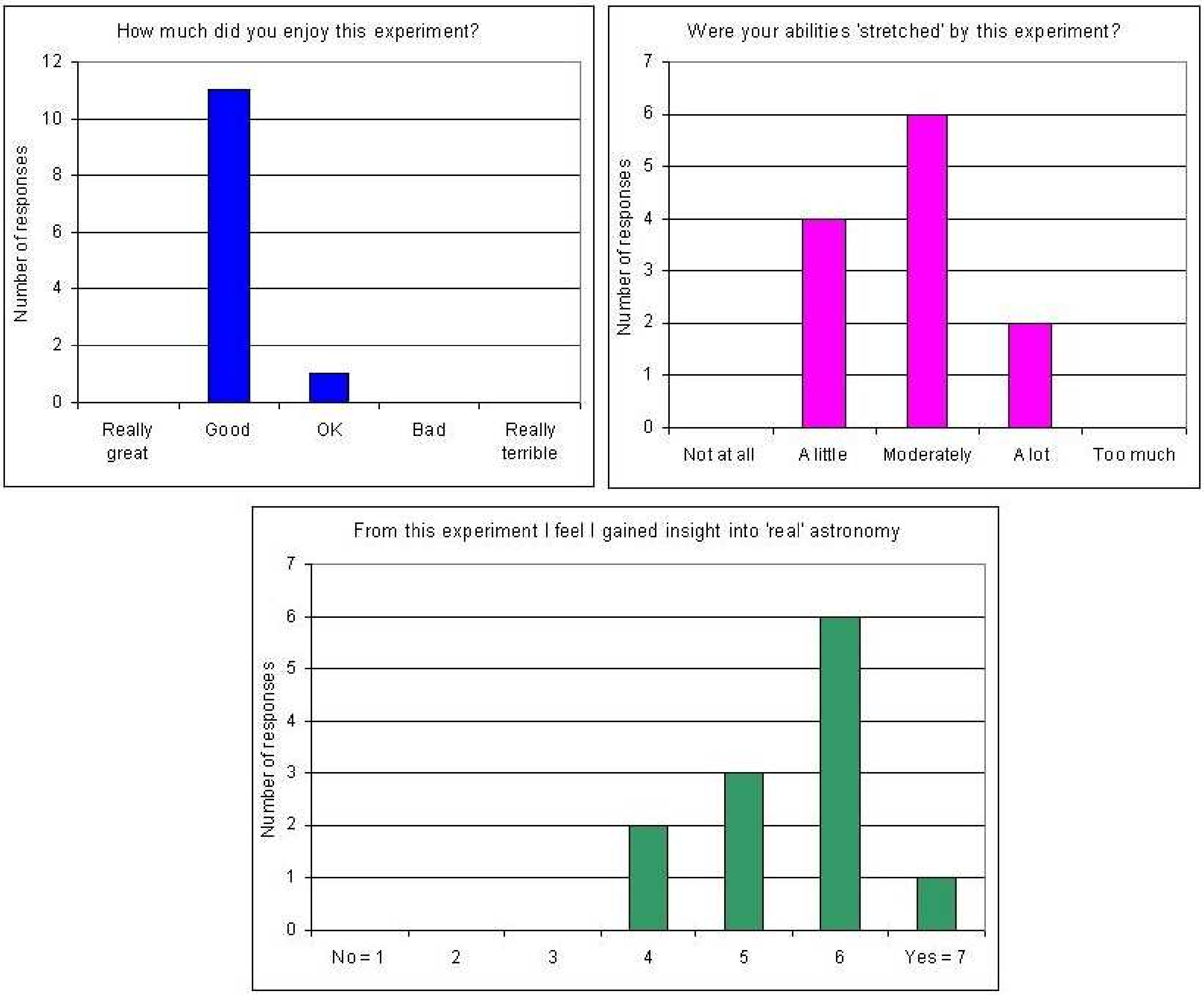

Figure 3. Student responses to the experiment

All the students exhibited some of the modes of scientific inquiry—for example, designing a technique, discussing implications of results, deciding how to handle data, pursuing "How do we know. . . ?" and "Why do we believe. . . ?" questions, forming concepts as a result of observations—that we try to develop in laboratory classes (Hegarty 1978; Arons 1993).

The style of script ensured that nothing was straightforward; even the brightest students had to take pause to think. This gave students real ownership of their results. They had followed the conceptual path of Eisenhauer et al. (2003) and understood what they had done.

6. CONCLUSION

Analysis using recent data to determine the distance to the black hole at the center of the Milky Way galaxy has been successfully undertaken by second-year university students in a three-hour laboratory session. Students were introduced to professional tools—Linux, GAIA and a FITS cube—and used them without difficulty. The style of the laboratory script required students to think conceptually, to make decisions about their analysis, and to be actively engaged. The experiment clearly demonstrated to them the relevance of Kepler's 17th-century laws to 21st-century data.

Any educators who would like to use the experiment with their own students are welcome to access the laboratory script and the data from http://www.physics.monash.edu.au/teaching/GC/. We ask that an e-mail be sent to the corresponding author susan.feteris@sci.monash.edu.au; we are interested in knowing if our work has been useful to others. We also ask that in any revised version of the script, the phrase "after the laboratory script described in Fitzgerald et al. 2008" be included, and that this article be included in the list of references.

URLs

GAIA: http://star-www.dur.ac.uk/~pdraper/gaia/gaia.html

NASA ADS: http://www.adsabs.harvard.edu/

Notes

Note 1: Max Planck Institute for Extraterrestrial Physics.

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