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A Construction to Determine the Azimuths and Times of Sunrise and Sunset

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Abstract

This article describes a paper-and-pencil construction to determine and graphically depict with reasonable accuracy the times and azimuths of sunrise and sunset on any day of the year at any location on Earth. The construction requires, as input, a date (or the Sun's declination) and the latitude and longitude of the location in question, and one needs only a compass, protractor, straight edge, and some dexterity to produce nice results. The method is visual and develops naturally along phenomenological lines, and so lends itself naturally to the classroom.

1. INTRODUCTION

Teachers at all levels do not face a shortage of tools to help them teach students about the seasons. A quick Web search turns up dozens of lesson plans, movies, animations, and computer programs that teachers can use to help students visualize and quantify the variations in the path of the Sun through the sky that give rise to the seasons. Many of these tools are quite good, especially some of the computer-based tools, which are very accurate and visually appealing.

Teaching students about the seasons is challenging. On one hand, a teacher may wish to focus mostly on qualitative work and emphasize symmetries in the seasonal variations. But astronomy is a quantitative science, and seasonal symmetries are clearly evident in numbers: sunrise/sunset time (available in the local newspaper) and azimuth. On the other hand, a teacher may wish to focus mostly on the quantitative. Yet when a teacher asks students to examine seasonal sunrise/sunset data closely, the work can become tedious and passive, and key seasonal symmetries can get lost in the detail.

Season-teaching tools, in this respect, present teachers with something of a dilemma. Simple tools are often too simple to be used to teach seasons quantitatively. And most computer-based tools, although quantitative and precise, are too passive and mask key seasonal symmetries with detail.

The construction described here balances qualitative reasoning and quantitative analysis. When students complete this activity, they (it is hoped) come away with accurate sunrise/sunset data and intuitively understand the daily and annual motions of the Sun on the celestial sphere.

2. METHOD OF THE CONSTRUCTION

The method of the construction is described next. The text and figures work through the specific case of sunrise and sunset on the summer solstice in Grand Haven, Michigan (43.0 degrees N, 86.2 degrees W), but one can easily generalize the procedure to any location and date.

It should be noted that this method derives in part from the work of Leighly (1970). His construction, however, is far more detailed than what is described in this article and is probably beyond the ability of most high school and beginning college students.

The first step in the construction is to draw an observer with his horizon and celestial meridian (Figure 1a). Recall that the celestial meridian is an imaginary line that runs from the north point on the horizon, through the point directly overhead—the zenith—to the South point on the horizon. The meridian semicircle on this diagram does not need to be drawn with any specific diameter, but it should be drawn carefully with a compass, and bigger is generally better.

Lay out the celestial axis—Earth's axis of rotation projected into the sky—on the meridian side-view. This line points from the observer O to the celestial pole P (very near the North Star in the northern hemisphere). The measure of angle NOP is equal to the observer's latitude ϕ. Next, mark the zenith Z directly above the observer O. Then locate the point T_e where the Sun crosses (transits) the meridian on the spring and fall equinoxes. The measure of angle POT_e is 90 degrees because the Sun is on the celestial equator on the equinoxes; alternatively, angle $ZOT_e = \phi$ or angle $SOT_e = 90 - \phi$.

To locate where the Sun transits the meridian on any other day of the year, one needs to know the Sun's declination δ. The Sun's declination is the number of degrees the Sun is north or south of the celestial equator. This angle varies between +23.5 degrees on the summer solstice and -23.5 degrees on the winter solstice, and is exactly zero on the equinoxes. The Sun's declination throughout the year is tabulated in a great many reference texts and Web sites (Note 1). It is often sufficient for instructional purposes, however, to compute sunrise/sunset on the summer and winter solstices when $\delta = +23.5$ degrees and -23.5 degrees, respectively.

Locate where the Sun transits the meridian on the day in question (the summer solstice, in the case of this example) by laying out segment OT so that angle $T_eOT = \delta$.

Last, mark a side view of the path of the Sun through the sky on the day in question. For the purposes of this construction, it is appropriate to assume that the Sun moves over the course of a day along a circular path parallel to the celestial equator centered on the celestial axis. So, construct segment Th parallel to segment T_eO. The Sun's path intersects the horizon at point h. Note that parallel lines can be constructed without using a ruler in a variety of ways.

The azimuths and times of sunrise are constructed from the meridian side-view by projecting the meridian side-view onto different planes using standard methods of projective geometry. Projecting the diagram onto different planes involves imagining how this diagram would look if viewed from different vantage points—an excellent mental exercise in its own right.

For azimuths, the diagram must be projected onto the plane of the observer's horizon. The end result is shown in Figure 1b, which is a view of the observer and his horizon from the vantage point of the zenith looking along segment ZO.

To begin, locate the observer in this top view by extending segment ZO a fair way down the page and drawing a horizon circle centered on this line with the same diameter as the meridian semicircle above. Draw EW and NS lines on the horizon circle that lie along, and perpendicular to, the segment ZO extension, respectively.

To project the sunrise and sunset points onto the horizon circle, drop a line from point h parallel to segment ZO. The sunrise and sunset points r and s are where this line crosses the eastern and western halves of the horizon circle, respectively. If the construction has been done carefully, the measure of angle rOE should equal the measure of angle sOW. This is a key symmetry of the seasons.

Azimuth is an angle measured from north clockwise around the horizon. The reader can verify with a protractor that in the case of the example worked here, the azimuths of sunrise and sunset are 57 degrees and 303 degrees, respectively. These results compare favorably with the actual sunrise and sunset azimuths on the summer solstice in Grand Haven of 56 degrees and 304 degrees.

Constructing the times of sunrise and sunset is only slightly more complicated: The meridian side-view must be projected onto the plane of the celestial equator. The end result is shown in the lower-right portion of Figure 1c. This view of the situation is from the vantage point of the celestial pole looking along segment PO.

Consider that to first approximation, over the course of a day, the Sun appears to move around the sky at a rate of 360 deg/24 hr. = 15 deg/hr. The Sun's daily path is a circle centered on the celestial axis and parallel to the celestial equator. In addition, our local clocks are set so that the Sun transits the meridian at about noon. (The Sun does not transit exactly at noon; see the next section.) Thus, looking from the celestial pole P along segment PO, one can imagine a great clock with the Sun marking the tip of the hour hand.

To construct this clock, first extend the celestial axis segment PO a fair way down and across the page. Then construct a line from point T parallel to the celestial axis. Draw a circle centered on the celestial axis with a radius equal to the perpendicular distance between T and the celestial axis. This circle is like the face of a tilted clock; it is noon when the Sun is at point T. Use Figure 1c as a visual guide. Note that the radius of this clock will be slightly less than that of the horizon circle constructed earlier. Label the center of this circle C, and draw 0–12 (midnight–noon) and 6–18 (6 AM–6 PM) lines on this circle, parallel and perpendicular to celestial axis, respectively.

To construct the times of sunrise and sunset, draw a line from point h on the meridian side-view parallel to the celestial axis so that it cuts across the clock circle. The sunrise and sunset times r and s are where this line crosses the AM and PM halves of the clock circle, respectively.

To derive sunrise and sunset times, measure clockwise angle 0Cr and angle 0Cs and convert them to times by dividing by 15 deg/hr. The reader can verify, in the case of the example worked here, that the sunrise and sunset times are 4:24 and 19:36. All times are standard time (EST, in the example worked here), not daylight time. During daylight savings time, our clocks are set ahead of astronomical time by one hour.

If the construction has been done carefully, then angle rC6 will equal angle 18Cs. Sunrise is before (or after, in the winter) 6 AM by the same amount of time that sunset is after (or before) 6 PM. This is another key symmetry of the seasons.

However, the mentioned sunrise and sunset symmetry is not often apparent in real sunrise and sunset data. This is so for several reasons.

The most obvious reason is that solar noon and civil noon, averaged over the year, occur at the same time only at places specifically located in their time zone. Observers too far east in their time zone find that solar noon occurs before civil noon, and observers too far west find that solar noon occurs after civil noon. One-hour-wide time zones span 15 degrees of longitude. And because Universal Time, or Greenwich Mean Time, is reckoned at 0 degrees longitude, solar noon and civil noon only on average occur at the same time at locations with a longitude that is evenly divisible by 15. In the continental United States, these time zone meridians are 75 degrees (Eastern Standard Time), 90 degrees (Central Standard Time), 105 degrees (Mountain Standard Time), and 115 degrees (Pacific Standard Time).

The sunrise/sunset construction described here can correct for this time zone error. Simply rotate the clock circle from its default orientation by an angle equal to the number of degrees of longitude that the location in question is east or west of its time zone's meridian. For example, Grand Haven is 11 degrees west of longitude 75 degrees, so solar noon occurs here on average about 11/15 = 0.73 hr. = 44 min. after civil noon. To correct for this, the clock circle must be rotated 11 degrees counterclockwise (the blue lines in Figure 1c).

The reader can verify that after this reorientation, sunrise and sunset are predicted to occur at 5:08 and 20:21. These times compare favorably with the actual sunrise and sunset times on the summer solstice this year in Grand Haven: 5:05 and 20:28.

3. DISCUSSION

The example worked in this article is a fairly typical sample: The construction is capable of predicting sunrise and sunset azimuths to a degree or so, and times to within about 10 minutes. It turns out that, owing to subtleties in the way our local clocks are synchronized to the Sun and in the way in which astronomical sunrise and sunset are reckoned, the method actually predicts sunrise and sunset more accurately than first seems apparent.

Before discussing briefly these subtleties, it stands to reason that the construction, to the extent that it has so far been described, allows students to predict sunrise and sunset with a precision that is suitable for most instructional purposes. Furthermore, the diagrams the students make to predict these events on different days of the year at different locations on Earth help them to appreciate key symmetries of the seasons and how date and location affect sunrise, sunset, and the path of the Sun through the sky. There may be no need to consider this method in more detail.

But an interested reader or a motivated student might still wonder why the construction method is not more precise. These details are provided for their benefit. For sunrise/sunset times, the construction relies on the fact that the Sun transits the meridian at precisely noon. The time zone correction—rotating the clock circle by the number of degrees of longitude that the location in question is east or west of its proper time zone meridian—is applied as compensation.

However, sundial makers worldwide have known for a long time that the Sun does not transit the meridian at the same time every day. Sometimes the Sun gets ahead of itself, so to speak, and sometimes it lags behind. A solar day—the length of time between successive meridian crossings of the Sun—is, on average and by definition, exactly 24 hours long, but every solar day isn't. Sometimes a solar day is a bit longer than 24 hours, sometimes a bit shorter. The solar day varies because Earth's axis of rotation is tilted with respect to the plane of its orbit (23.5 degrees, the obliquity of the ecliptic) and because Earth orbits the Sun along an elliptical path. A complete explanation is beyond the scope of this article.

One interesting observable consequence of this variation, one that many are surprised to find (and students often discover this), is that the summer solstice is not the day of the earliest sunrise or of the latest sunset. Nor is the winter solstice the day of the latest sunrise or of the earliest sunset. This is noted by West and Keith (1996) and Garstang (1996).

A small correction must be applied to sundials to convert from the apparent solar time that they indicate to mean solar time to which our clocks are synchronized. The correction, amounting to as much as +15 min. (in February) to -16 min. (in November), is called the equation of time. More information about the equation of time can be found elsewhere (Note 2).

The sunrise/sunset construction is like a sundial: The times it predicts need to be corrected by the equation of time. On the summer solstice, the equation of time is about -2 min., which means that the predicted sunrise and sunset times are 2 min. too early. The corrected times of sunrise and sunset on the summer solstice in Grand Haven are 5:10 and 20:22. Thus, it is apparent that the predicted sunrise is 5 min. after the true sunrise, and the predicted sunset is 5 min. before the true sunset. The symmetry in the error is pleasing.

The remaining error in the times of sunrise and sunset, along with the error in the predicted azimuths, is due to the way sunrise and sunset are reckoned in practice.

Astronomical sunrise and sunset occur when the very top of the Sun first slips above, and finally slips below, the horizon, respectively. But the construction method described in this article tracks the position of the center of the Sun (this is the point to which the Sun's declination refers). On the solstices, especially at high latitudes, the Sun's path through the sky at sunrise and sunset makes an acute angle with the horizon. The Sun slides along the horizon as it rises and sets. And although the Sun's apparent diameter is only about 0.5 degrees, the Sun's center always rises after, and sets before and more to the south than, its upper edge.

Furthermore, because of refraction in the atmosphere, we can see the Sun even when it is below the true geometric horizon. These final two complications could be included in this construction, but doing so would change its flavor a bit too much.

One final remark about the construction: It is left as an exercise for the reader to work through the trigonometry of the diagrams to verify that the azimuth of sunrise is given by

$$\cos(NOr) = \sin \phi / \cos \delta,$$

and that the time of sunrise expressed as an angle (on a time zone meridian) is given by

$$\cos(0Cr) = \tan\phi \tan\delta .$$

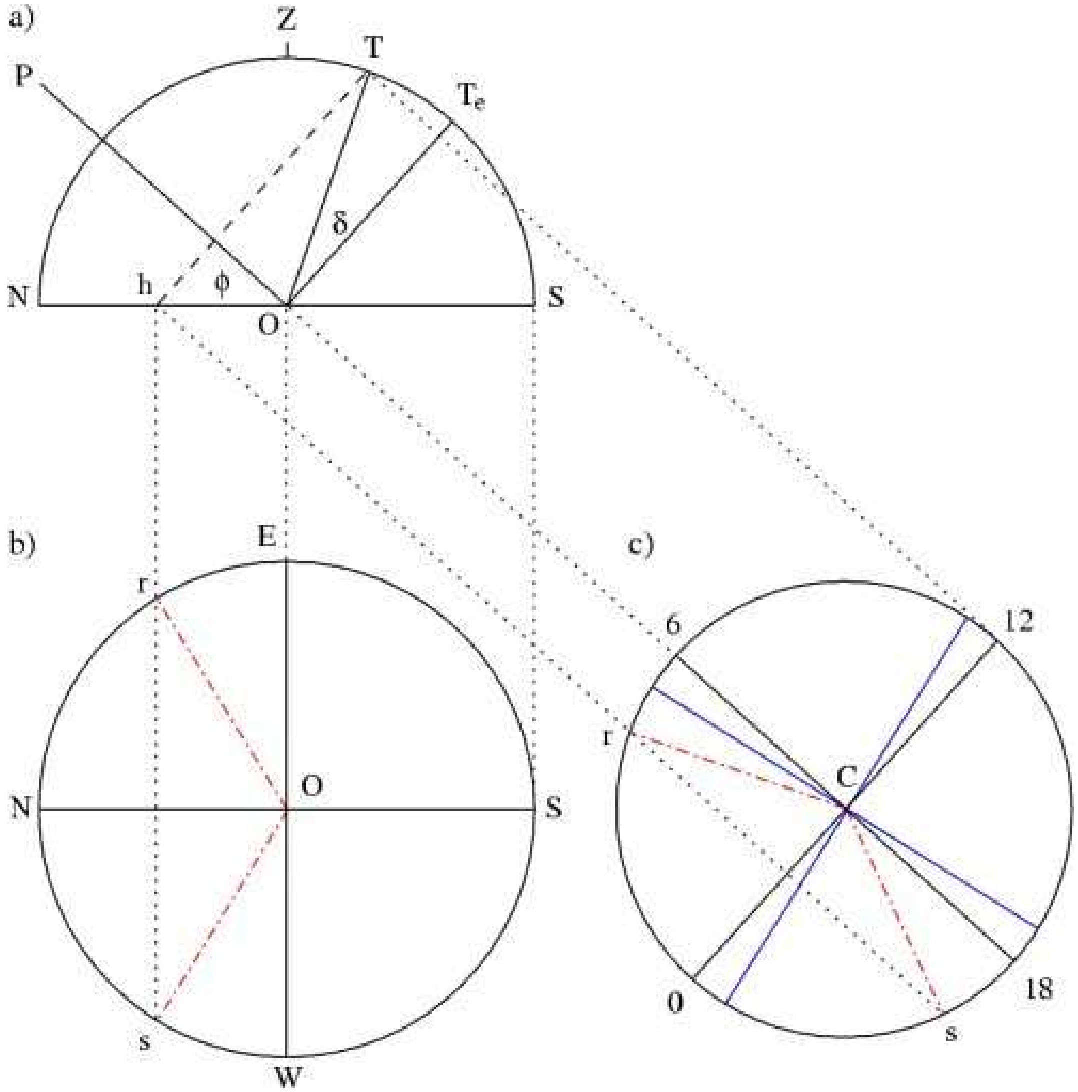

Figure 1. The construction described in detail in this article. (a) A side view of an observer with his horizon and meridian. ϕ is the observer's latitude, and δ is the Sun's declination. T and T_e are the points where the Sun transits the meridian on the equinoxes and the day in question. (b) A projection of the situation onto the observer's horizon: the sunrise and sunset points on the observer's horizon are r and s. (c) A projection of the situation onto the plane of the celestial equator: the sunrise and sunset times are points r and s. The gray lines reorient the sunrise/sunset clock if the location is not on a time zone meridian.

4. CONCLUSION

Teachers at the high school and college levels can use the construction described in this article as a tool to help them teach the seasons. In some classes, it may be sufficient to use only the azimuth part of the construction in class. Other classes might benefit from both the azimuth construction and the time construction.

Note also that the time construction can be done to several levels of precision: (a) predict sunrise/sunset times on a time zone meridian, (b) apply the off-time-zone-meridian correction, and (c) apply the equation-of-time correction. A teacher should consider this before using the construction as a season teaching tool. Note also that if one wishes only to determine the length of time that the Sun is above the horizon on a particular day—for example, on the summer solstice—corrections (b) and (c) are not important, and the construction gives good results in its default form.

When students go through this construction in quantitative detail for a particular location, and then browse a bulletin board of diagrams constructed by their peers for different locations and/or different days of the year, they come away with a tangible and accurate way to visualize the variations in the path of the Sun through the sky that give rise to the seasons.

NOTES

Note 1. For example, http://www.wsanford.com/~wsanford/exo/sundials/DEC_Sun.html.

Note 2. For example, http://en.wikipedia.org/wiki/Equation_of_time

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84 - 91