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Studying the Transit of the Sun Using Shadows

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Abstract

This article describes how to accurately determine the time of the Sun's transit and the north-south direction by observing a stick's shadows. The same observations can be used to determine the latitude and longitude of the location of the observation. Calculating the latitude and longitude is particularly simple for certain dates.

1. INTRODUCTION

The shadows of a stick can tell us a lot about the motions of the Earth and Sun. Jackson (2004) discussed the use of shadows to study the transit of the Sun, the north-south direction, and the seasonal changes of the shadows. In this article, I discuss alternative ways of observing the transit of the Sun and other activities using shadows. The discussions and diagrams are for the northern hemisphere, but with appropriate conversions, these activities are also applicable in the southern hemisphere.

2. OBSERVING THE SHADOWS

When the Sun is at transit at noon, shadows have their shortest lengths for that day. The direction of shadows at transit is north-south, but it is difficult to judge when this happens because the Sun is not necessarily at transit at clock noon. Jackson's method relies on finding the time of transit by reading sunrise and sunset times in the local newspaper, but this information is not always available. Although several Web sites provide online calculations of the sunrise and sunset times, including the U.S. Naval Observatory (http://aa.usno.navy.mil/data/docs/RS_OneYear.html) and Her Majesty's Nautical Almanac Office (http://websurf.nao.rl.ac.uk), they are not needed to determine when the Sun is at transit. The procedures for determining the time of transit are outlined below.

Lay out a large sheet of paper on the ground or on a drawing board on the ground. Erect a stick with a pointed end upward. The stick should be vertical. A simple way to hold the stick upright is to use Blu-tack or plasticine. (See Figure 1.)

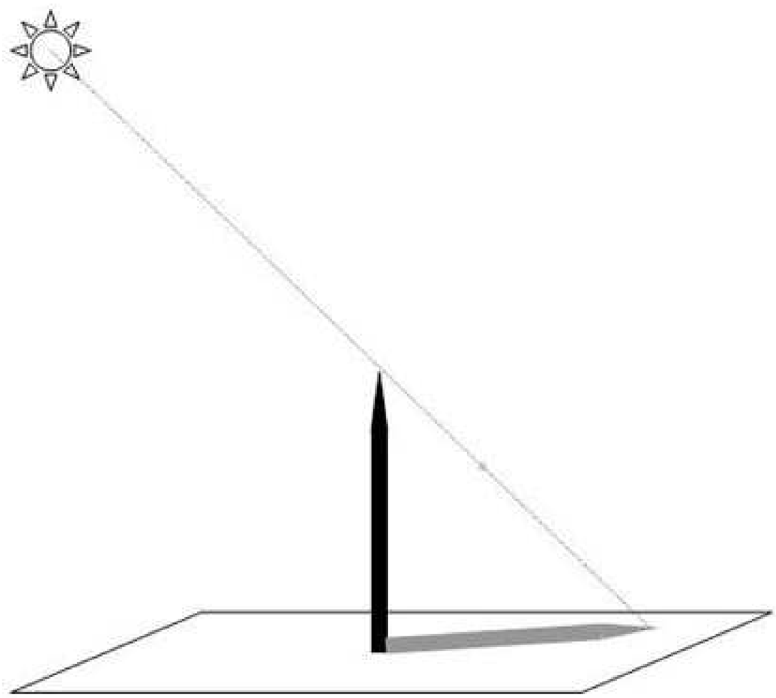

Figure 1. Set-up for observing the shadows of a stick.

The size of the paper depends on the duration of the shadow observation. If you start very early in the morning and finish very late in the afternoon, you probably need a large sheet of paper. The size of the paper needed also depends on the height of the stick. For example, if you use a 25-cm bamboo skewer (available in supermarkets for BBQs) as the stick, the typical paper size is about 75 cm by 60 cm, or two A-3 size sheets of paper joined together. Preferably, the observation should start at least two hours before noon. Have the watch calibrated with the local standard time, mark the position of the tip of the stick's shadow on the paper, and note the time. Take a measurement about every 15 to 30 minutes. When the Sun is near transit, take measurements more frequently. You can stop taking measurements about two hours after transit. Another way to judge when to stop is to see whether there are enough measurements to make the track of the shadows look symmetrical at about the transit time. With the position of the paper still fixed, remove the stick. Connect all of the marked positions of the shadow with a smooth line (dotted lines in Figure 2). Draw a circle with the stick location in the center. The radius is arbitrary as long as the circle intersects the curve of the shadow at two points--A1 and B1 in January, A3 and B3 during equinoxes, and A6 and B6 in June--on either side of the transit for places north of the Tropic of Cancer (see Figure 2).

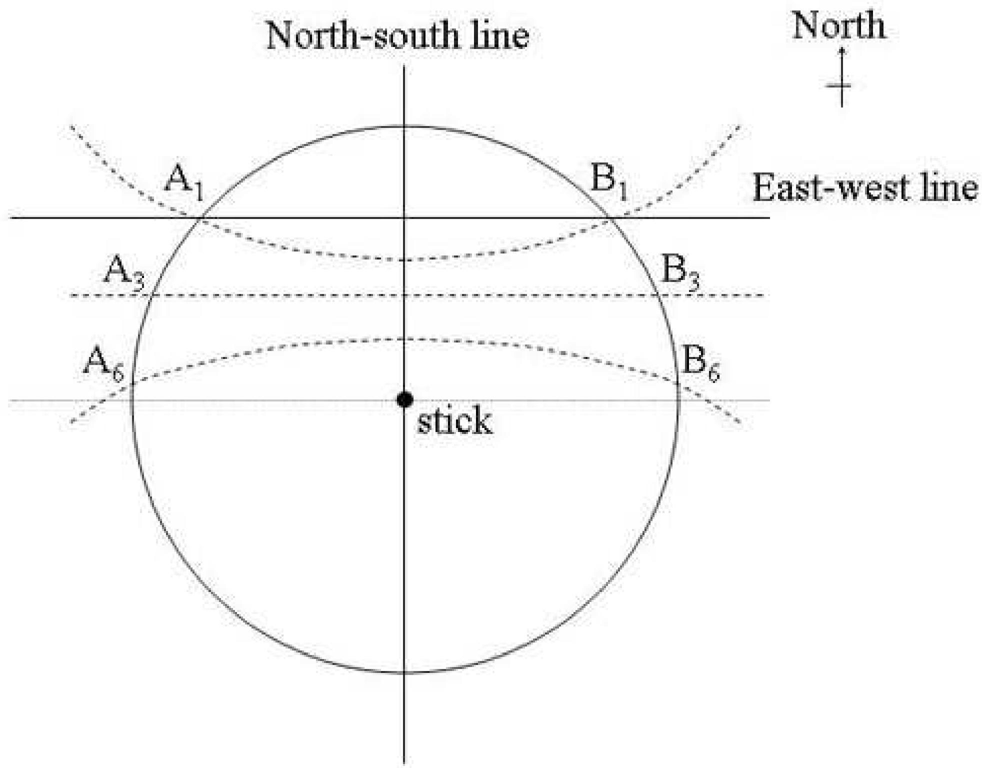

Figure 2. Tracks of the shadows (dotted lines) in January, equinoxes, and June (from top to bottom) for places north of the Tropic of Cancer.

The line joining the intersecting points is in an east-west direction. You can then draw a perpendicular line through the center of the circle. This is the north-south line. With this method, you do not have to know the local transit time of the Sun. The time when the transit occurs can be determined using interpolation. If you mark the positions of the shadows frequently enough near transit, the time of the transit can be determined quite accurately.

The accuracy of the east-west direction, hence the north-south direction, can be increased if the intersections of the circle and the shadow of the track are farther apart. A larger scale helps because there are always errors in marking the shadow positions, drawing the track of the shadows, drawing the circle, and judging the intersecting points. Let the overall uncertainty in locating the intersecting points be y. The uncertainty of the east-west direction is about arctan(y/x), where x is the separation of the intersecting points. Because y is more or less the same, increasing x reduces the uncertainty of the direction. (See Figure 3.)

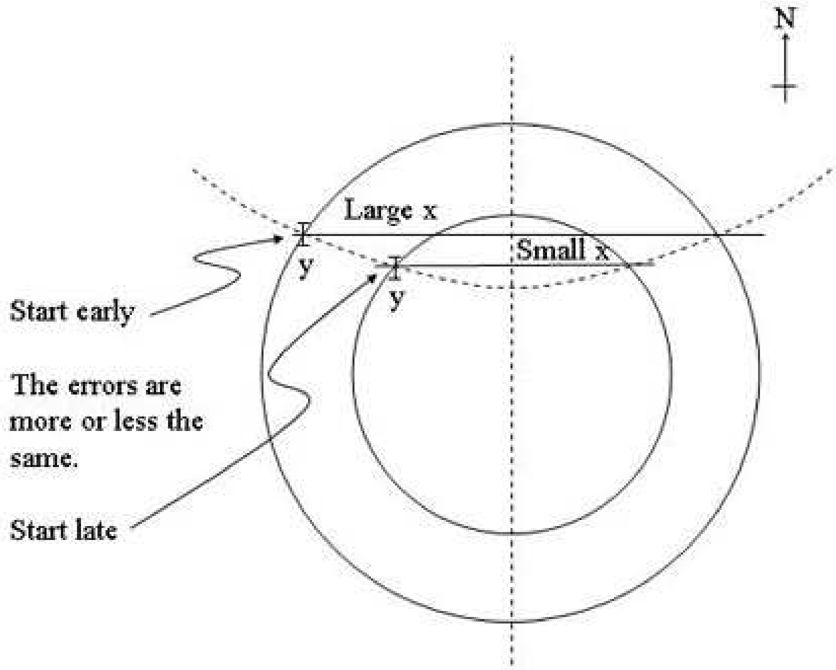

Figure 3. Increase the accuracy of the east-west direction by maximizing the separation, x, of the intersecting points. The errors, y, are greatly exaggerated in the diagram.

If the measurement starts early in the morning and ends late in the afternoon, you can draw a bigger circle to intersect the track of the shadows and hence improve the accuracy. Once the north-south direction is determined, the length of the shadow at transit can be measured at leisure and with better accuracy than if it is done while the observations are in progress.

The observation of the shadows usually requires a wide open area on a sunny day so that the stick can cast shadows throughout the day. However, it may happen that it is sunny when you start the observation in the morning, and becomes cloudy in the afternoon, or the opposite. You have about half of the symmetric curve of the shadow track, and the circle intersects the curve at only one point. As long as the curve includes the transit segment, you can still make use of your observational data even though the curve is asymmetric and incomplete. Because the curve must be symmetrical and the line must join smoothly with its mirror image, we can use a mirror to find the north-south direction. If the mirror is not along the north-south direction, the observed curve and its mirror image will have either an outward pointing cusp (Figure 4a) or an inward pointing cusp (Figure 4b).

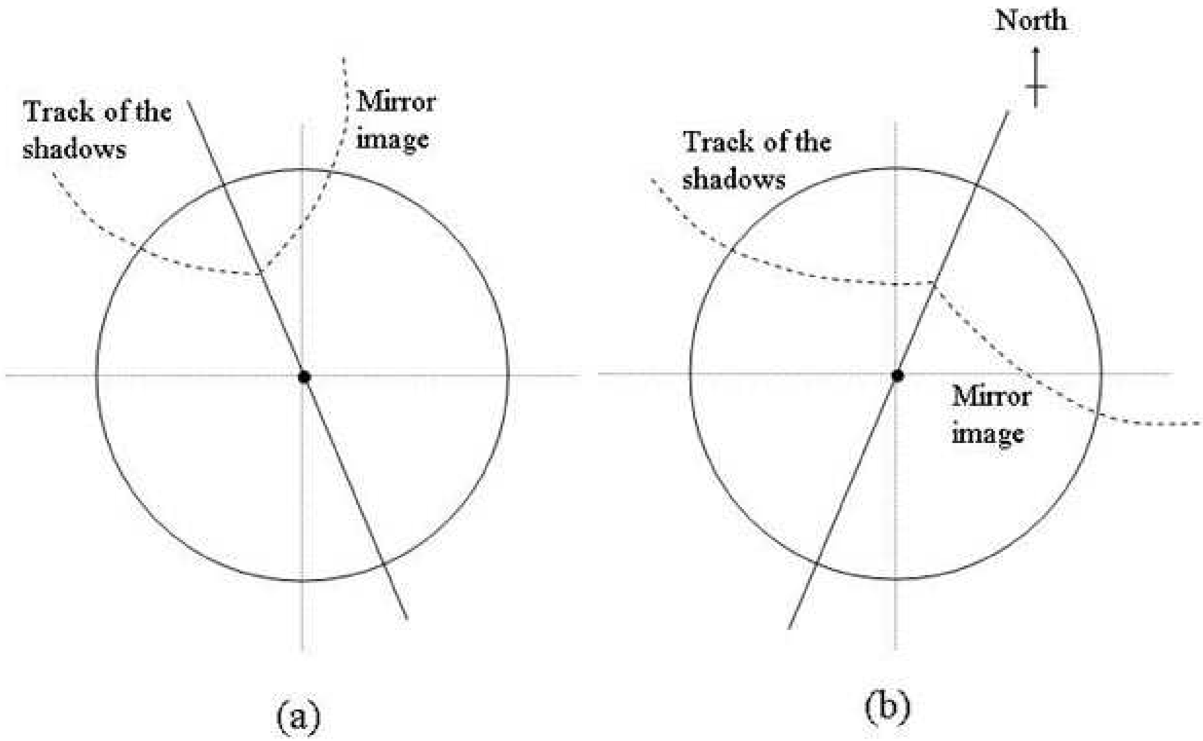

Figure 4(a). Mirror points to the northwest direction. **Figure 4(b).** Mirror points to the northeast direction.

This method is not very sensitive when the mirror is near the north-south direction; the uncertainty could be up to 1 to 2 degrees. However, this procedure is still better than using a magnetic compass alone. The magnetic declination for most of the Earth is much larger than this uncertainty (NOAA National Geophysical Data Center, 2004). The method of using the mirror image is also applicable to circumstances in which continuous sunshine from morning until afternoon is not possible. In Hong Kong, many schools are surrounded by high-rise buildings or next to a hill slope. Sunlight reaches the schools for only about half the day. This method allows schools to run the activity in any schoolyard without the logistical problems such as transportation and personal safety. For safety reasons, it is advisable to use plastic or acrylic mirrors instead of glass mirrors for this activity. Plastic mirrors are available in hardware stores or mirror stores, or they can be ordered online. Unlike glass mirrors, plastic mirrors are easier to work with and can be cut to any size that suits your activity, without special tools or techniques.

3. THE LATITUDE

The stick shadows can also be used to find the latitude and longitude of the location where the stick stands. To find the latitude, in general we need to know the altitude of the Sun at transit (angle A) and the declination of the Sun (angle D). The latitude (angle B) is given by

Latitude = 90 degrees - (angle A + angle D)

The declination of the Sun can be found from the Astronomical Almanac (United States Naval Observatory Nautical Almanac Office, published yearly) or from the Web version of MICA (United States Naval Observatory, 2003). The measurement is particularly simple if it is taken in equinoxes because the declination of the Sun is zero on these days. (See Figure 5.)

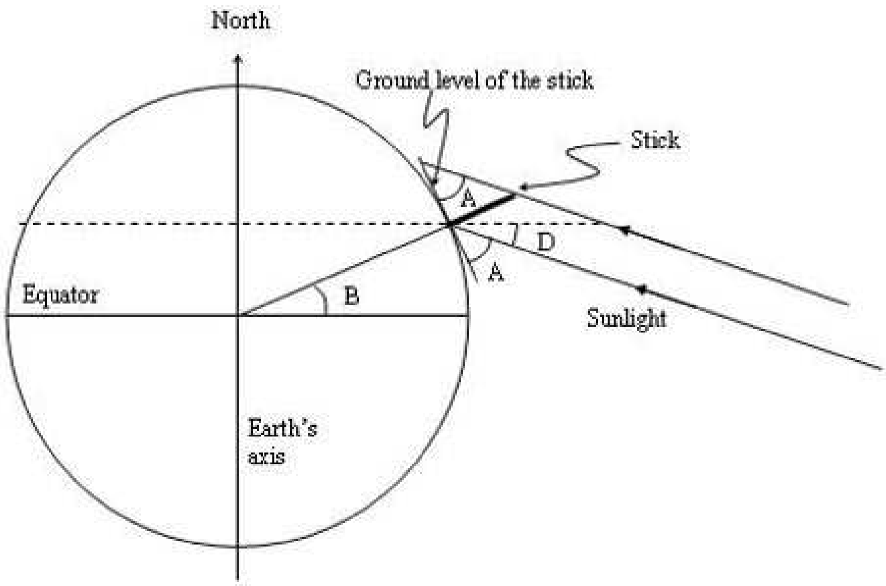

Figure 5(a). Sunlight direction and latitude.

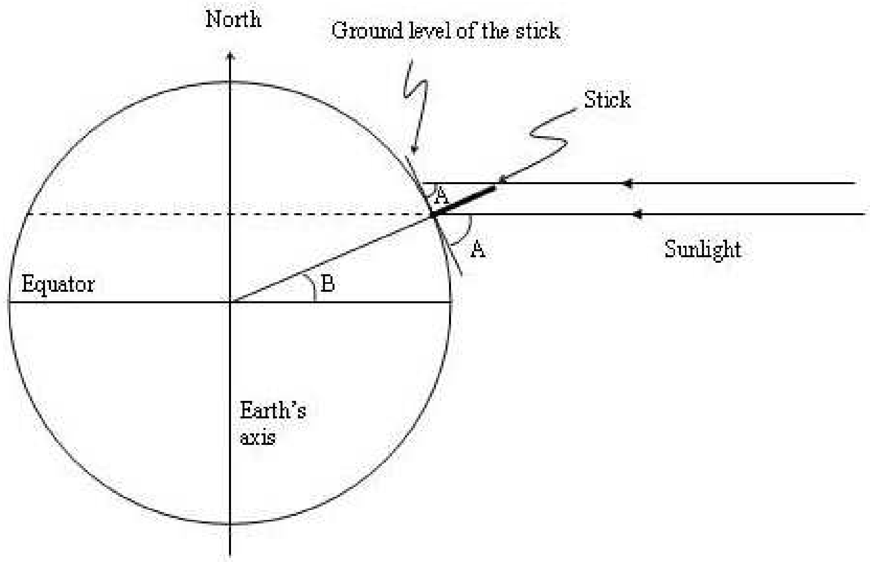

Figure 5(b). Sunlight in the spring and autumn equinoxes.

4. THE LONGITUDE

From the time of transit of the Sun, we can find the longitude of the stick. The local time--as measured by the position of the Sun--at one longitude is different from the local time at another longitude. Life would be very confusing if we operated on local time. My watch would have a different time from your watch if we were at different longitudes, even in the same city. The use of time zones avoids this confusion. Standard longitudes are chosen to be at 0 degrees, 15 degrees east/west, 30 degrees east/west, and so on. Places within + or - 7.5 degrees of the standard longitudes all use the same time, which is called mean solar time and is the same as the local time at the standard longitudes. (This simple rule is sometimes modified to take account of geographic and political boundaries.) Of course, at places east of the standard longitude of the time zone, the Sun occurs earlier (e.g., 11:45 a.m.) than at places west of the standard longitude of the time zone (e.g., 12:20 p.m.). Because four minutes difference in time translates to 1 degree in longitude, the difference in time between the transit of the Sun at any specific place and the standard longitude for the same time zone can be used to find the longitude of the place where the transit was observed.

Unfortunately, things are a bit more complicated than what I just wrote. The Sun does not move uniformly in the sky. Even if we are at the standard longitude, the Sun transit time does not always happen exactly at 12 noon. It may happen a bit earlier or a bit later depending on the seasons. The difference between the actual transit time (local apparent solar time, LAST) and the mean transit time (local mean solar time, LMST) is called *equation of time* and can be up to 16 minutes.

Equation of time = LAST - LMST

The equation of time can be found from the "figure 8," also called analemma, usually printed on the surface of a globe. It is also tabulated in some Web sites (e.g., http://www.analemma.com/Pages/Summation/SummationMath/EOTv12.xls) or calculated according to the procedures in the Astronomical Almanac. The time recorded by the shadow is the LAST; correction is then applied to obtain the LMST. Once the LMST is found, it is straightforward to convert the time difference to longitude.

When the correction happens to be zero, the calculation is very simple. There are four times each year that the corrections are zero. These four dates are April 16, June 15, September 2, and December 25. If local solar transit is measured within 1 to 2 days on these four dates, the local apparent solar noon is just the local mean solar noon. For example, if the LMST at transit is 11:45 a.m., the longitude of the stick is

Longitude = longitude at standard meridian + (12:00 - 11:45)/4 for the eastern hemisphere.

Longitude = longitude at standard meridian - (12:00 - 11:45)/4 for the western hemisphere.

If the LMST at transit is 12:20 p.m., the longitude of the stick is

Longitude = longitude at standard meridian - (12:20 - 12:00)/4 for the eastern hemisphere.

Longitude = longitude at standard meridian + (12:20 - 12:00)/4 for the western hemisphere.

Students find it amazing that without a GPS, they can locate their latitude and longitude with simple tools and calculations. This is a skill that one would find very useful if one were stranded in a remote place on Earth.

Acknowledgment

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